# TWO

# The Celestial Sphere

### *Basis in Observation*

To a naive observer it is by no means obvious that the sky has the shape of a dome or hemisphere. Indeed, the Egyptians in their art often represented the sky by the sky goddess, Nut, arched over the land, often, though not always, with her back flattened (see fig. 2.1). And the Egyptian hieroglyph for *sky* () is reminiscent of the flat roof of a long, low building.[1]

Although the stars themselves suggest no particular shape for the heaven, their motions do suggest a sphere: the Moon, stars, and planets are seen moving on parallel circles, climbing up together from the eastern horizon, crossing the sky, and going down together in the west. Even more suggestive are the circumpolar stars, which can be seen all night long, moving in circles about the celestial pole. Ptolemy pointed to exactly these facts of observation in trying to explain how his remote predecessors had come to the idea of a spherical heaven.[2]

"*The heaven is spherical and moves spherically.*"[3] This is the most fundamental assumption of Greek astronomy. Many Greek astronomical texts begin with it or something like it.[4] Although this view often was supported with arguments of a philosophical or even mystical nature, it is actually suggested by observation of the sky. Aristotle regarded the sphere of the fixed stars as a real, material sphere, and all later astronomical writers, including Ptolemy, followed him in this.

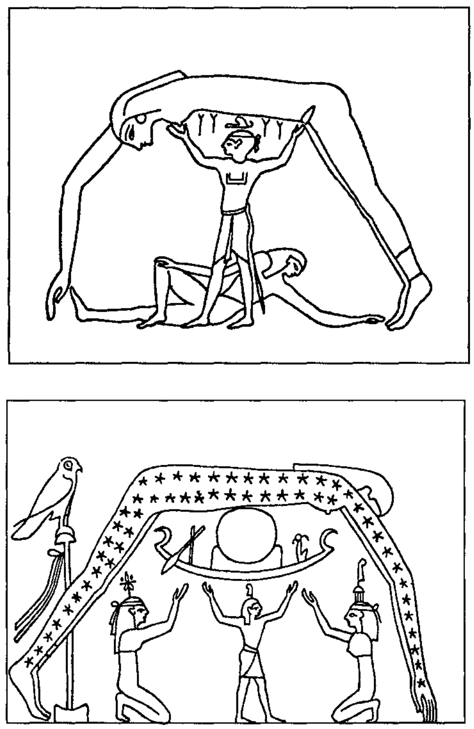


FIGURE 2.1. The Egyptian sky goddess, Nut. *Top*: Nut supported by the air god, Shu. Beneath them lies the earth god, Seb. *Bottom*: Shu supporting the boat of the sun god beneath the sky goddess, Nut. From Budge (1904).

### *Eudoxus and Aratus on the Sphere*

The idea of a spherical cosmos can be attributed to sixth- and fifth-century B.C. philosophers such as Pythagoras and Parmenides. But the first figure in whom we see a clear and complete understanding of the celestial sphere is Eudoxus of Cnidus (ca. 370 B.C.). Eudoxus was the author of a number of astronomical works, including a star calendar and a treatise on the eight-year luni-solar cycle. He is the likely, if unproved, source of the earliest known measurement of the circumference of the Earth, mentioned in passing by Aristotle in *On the Heavens*. Among his other writings, Eudoxus is known to have composed two books on the celestial sphere, called the *Phenomena* and the *Mirror*. These books, which apparently differed little from one another, contained systematic descriptions of the constellations and their relative positions on the sphere.

Not one of Eudoxus's works has survived. But in the case of the *Phenomena* we have a paraphrase of one of them, for Eudoxus's description of the night sky inspired the poet Aratus of Soli to produce a versified version about a century later. The verse *Phenomena* of Aratus proved to be very popular: commentaries were written on it, it was on several occasions translated into Latin, and it even inspired sculptors and other artists to treat astronomical themes. Among the many commentators on Aratus was Hipparchus of Bithynia (ca. 150 B.C.), the most creative astronomer of the Hellenistic age. Hipparchus still had access to Eudoxus's original prose *Phenomena*. In his *Commentary* Hipparchus makes a painstaking examination of the works of his two predecessors. Often, he finds them inexact or mistaken about positions of the stars and constellations. Hipparchus tells us, however, that we ought not blame Aratus, who was a poet and not an astronomer, and who was in any case only following Eudoxus, but Eudoxus, as an astronomer, must be held accountable for the errors.[5] Hipparchus makes Aratus's dependence on Eudoxus clear by quoting parallel passages from the two versions of the *Phenomena*. Thus, although Eudoxus's *Phenomena* has not come down to us, we may safely assume that its astronomical content is reflected in the poem of Aratus.

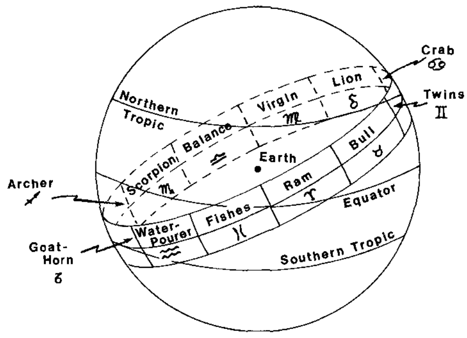



FIGURE 2.2. The celestial sphere and the signs of the zodiac.

| *Symbol* | *Greek* | *Latin* | *English* |
|---|---|---|---|
| ♈ | Krios | Aries | Ram |
| ♉ | Tauros | Taurus | Bull |
| ♊ | Didymoi | Gemini | Twins |
| ♋ | Karkinos | Cancer | Crab |
| ♌ | Leōn | Leo | Lion |
| ♍ | Parthenos | Virgo | Virgin |
| ♎ | Zygos | Libra | Balance |
| ♏ | Skorpios | Scorpius | Scorpion |
| ♐ | Toxotēs | Sagittarius | Archer |
| ♑ | Aigokerōs | Capricornus | Goat-Horn[ed creature] |
| ♒ | Hydrochoös | Aquarius | Water-Pourer |
| ♓ | Ichthyes | Pisces | Fishes |

Aratus begins with descriptions of the constellations and their positions on the sphere, along with stories and legends about them. Then he briefly describes the four principal circles of the celestial sphere: equator, zodiac, and the two tropic circles.[6] The image that emerges can be represented as in figure 2.2. Girding the celestial sphere are the three parallel circles of the equator and the two tropic circles. The fourth important circle is the slanted zodiac, which lies athwart the tropics. Although the Greek heaven of figure 2.2 still bears mythological images, it represents a radical break with traditional cosmologies, typified by the Egyptian images in figure 2.1. In introducing the theory of the celestial sphere, the Greeks took a decisive step toward geometrizing their worldview.

Aratus goes on to list the constellations that lie on each of the four circles. The northern tropic circle passes through the heads of the Twins (Gemini) and the knees of the Charioteer (Auriga), passes just below Perseus, but straight across Andromeda's right arm above the elbow. Also lying on the northern tropic are the hoofs of the Horse (Pegasus), the head and neck of the Bird (Cygnus), and the shoulders of Ophiuchus. The Virgin (Virgo) is a little south of the tropic and does not touch it, but both the Lion and the Crab (Cancer) are squarely on it. This detailed description would enable the reader to visualize the tropic of Cancer in the night sky. Aratus gives similar lists of the constellations lying on the equator and tropic of Capricorn. For the zodiac—the fourth major circle—the list consists of the familiar twelve zodiacal constellations.

A note on terminology: it is important to distinguish between *zodiac* and *ecliptic*. The Greek astronomers thought of the zodiac as a band of finite width, as in figure 2.2, rather than as a vanishingly thin circle. The circle that runs down the middle of this zodiacal band is the ecliptic (the Sun's path), which the Greeks called *the circle through the middles of the signs.* The Moon and the planets move nearly along the ecliptic, but the Moon may wander north or south of it by as much as 5°. The maximum latitudinal wanderings of the planets range from about 2° in the case of Jupiter to about 9° in the case of Venus. The zodiac was conceived of as a band wide enough to encompass these wanderings.

## *Fundamental Propositions of Greek Astronomy*

From the time of Eudoxus on, Greek astronomy was based on five fundamental propositions:

1. The Earth is a sphere,
2. which lies at the center of the heaven,
3. and which is of negligible size in relation to the heaven.
4. The heaven, too, is spherical
5. and rotates daily about an axis that passes through the Earth.

We have discussed propositions 4 (sphericity of the heaven, in the present section) and 1 (sphericity of the Earth, in sec. 1.9). In section 1.6, we examined the ancient debate over proposition 5 (rotation of the heaven). In considering the two remaining propositions, we will examine some of the arguments offered by Ptolemy in *Almagest* I.

*That Earth Is in the Middle of the Heaven* Suppose, says Ptolemy, that the Earth is not at the center of the celestial sphere. Then it is either

(a) off the axis of the sphere but equidistant from the poles,
(b) on the axis but farther advanced toward one of the poles, or
(c) neither on the axis nor equidistant from the poles.

Let us examine case (a). In figure 2.3, the Earth lies off the axis of the celestial sphere, but at equal distances from the two celestial poles. In this case there will be trouble with the equinoxes. Let an observer be at *A* on the Earth's equator, with horizon *YAW*. At the time of the equinox, the Sun lies on the celestial equator and therefore runs around circle *WXYZ* in the course of one day. The observer at *A* will see the Sun above the horizon only for the short time the Sun requires to run arc *YZW*, and the Sun will be below for the long time it takes to travel arc *WXY*. But this contradicts the observed fact that, at equinox, the period of daylight is equal to the period of darkness at all places on Earth.

Now consider case (b), in which the Earth is on the axis of the universe but nearer one of the poles. Then everywhere (except at the Earth's equator) the plane of the horizon will cut the celestial sphere into unequal parts, which is contrary to observation, since one half of the sphere is always found above the horizon (fig. 2.4). And it is not possible to advance to case (c) since the objections to (a) and (b) would apply here also.

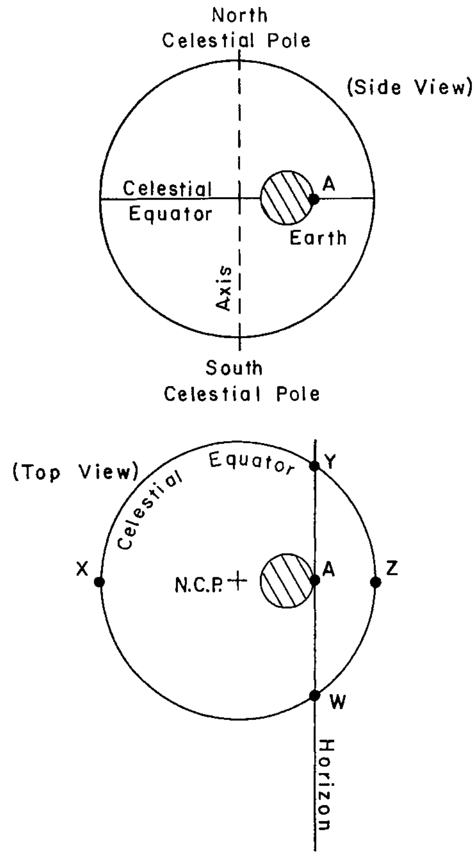



FIGURE 2.3.

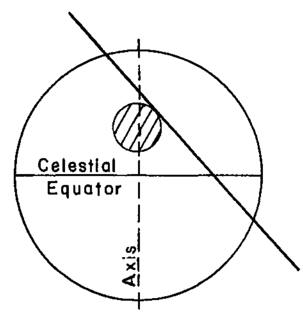



FIGURE 2.4.

*That the Earth Is a Mere Point in Comparison with the Heaven* In the first place, says Ptolemy, if the Earth had an appreciable size compared with the celestial sphere, the same two stars would appear, to observers at different latitudes, to have different angular separations. For example, in figure 2.5, observers at *D* and *E* will measure different angular separations between the stars *F* and *G*. That is, angles *FDG* and *FEG* are not the same. Further, star *G* will appear brighter to the observer at *E* than to the observer at *D*. But all of this is in contradiction to the facts, for the stars actually appear the same in the different latitudes.

Second, the tips of shadow-casting gnomons can everywhere play the role of Earth's center, which could not be the case if the Earth had any appreciable size. For example, as in figure 2.6, let gnomon *AB* be perpendicular to the terrestrial meridian *CG*. At noon on the winter solstice, the Sun is at *H* and produces the shadow *BD*; at noon on the equinox, the Sun is at *I* and produces shadow *BE*; and finally, at summer solstice, the Sun, at *J*, produces shadow *BF*. Now, at any place whatever on the Earth it is found that angle *JAI* = angle *IAH*, roughly 24°. Thus, the tip *A* of the gnomon may always be taken as the center of the sphere of the Sun's motion. But if this is true everywhere, the Earth must be very small compared to the celestial sphere. The fact that, for any place on Earth, the tip of the gnomon can be treated as the center of the cosmos would have been familiar to any of Ptolemy's readers who had studied the techniques of constructing sundials. We make use of this fact ourselves, in section 3.2, where we study the construction of Greek and Roman sundials.

We have expanded some of Ptolemy's arguments and illustrated them with figures for the sake of greater clarity. These arguments were not, however, original with Ptolemy, since some of them were used by earlier writers, for example, Euclid and Theon of Smyrna. Indeed, the essential arguments concerning the heaven and the Earth's place within it were already hundreds of years old by the second century A.D., when Ptolemy wrote. Ptolemy merely presented the case with greater thoroughness and organization. These arguments remained the common stock of all astronomers down to the Renaissance.

*Critique of the Ancient Premises* In general, the Greek astronomers believed in the literal truth of all five propositions. In an introductory astronomy course, the teacher would probably have marched his students through the five propositions, giving ample proofs of each, the proofs being based not only on appeals to observation but also on physical and philosophical argument.

But from our perspective, while some of these propositions may be regarded as rigorously proved, others only reflect a point of view. In particular, propositions 1 and 3 (sphericity and smallness of the Earth) are not only provable but actually were proved in antiquity. Proposition 2, which places the Earth

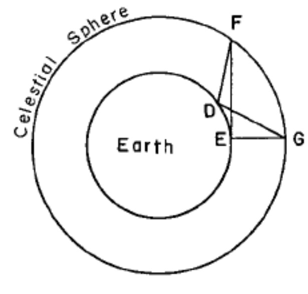



FIGURE 2.5.

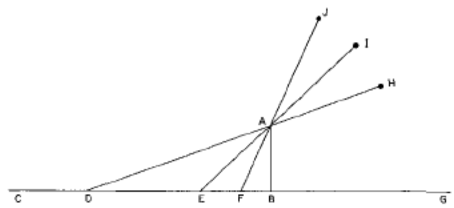



FIGURE 2.6.

at the center of the heaven, rests partly on empirical evidence and is partly conventional. Certainly, the axis of the daily rotation must pass through the Earth. But, granted this, as long as the stars are very far away, it can make no difference whether the Earth is exactly in the middle of things or not. Every observer, whether on the Earth, the Moon, or Jupiter, can legitimately treat his or her own home as the center of the universe (as far as appearances are concerned). Proposition 4, that the heaven is spherical, is wholly conventional. Because the stars are very far away from us, it makes no difference whether they all lie on a single spherical surface or not. But we will not get into trouble by assuming that they do. Proposition 5 also reflects a point of view. We may say with equal validity that the heaven rotates once a day from east to west or that the Earth rotates from west to east.

## 2.2 SPHAIROPOIIA: A HISTORY OF SPHERE-MAKING

### *Some Representative Globes and Armillary Spheres*

The most ancient known celestial globe is a large stone sphere supported by a statue of Atlas, in the Museo Nazionale at Naples. This statue, transferred to its present location from the Farnese Palace in Rome, is called the Farnese Atlas. The globe is a Roman copy (first or second century A.D.) of a Hellenistic original made perhaps several centuries earlier.

The Farnese globe is shown in figure 2.7. The Earth, not represented, would be a tiny sphere located inside the celestial sphere. Part of the globe is obscured by the hand of the statue of Atlas that supports it. In figure 2.7,

FIGURE 2.7. The Farnese globe. This ancient marble celestial sphere was supported by a statue of Atlas, whose hand is visible on the globe. From G. B. Passeri, *Atlas Farnesianus . . .*, in Antonio Francesco Gori, *Thesaurus gemmarum antiquarum astriferarum*, Vol. III (Florence, 1750). By permission of the Houghton Library, Harvard University.

the zodiac belt is the triplet of rings arching across the upper part of the globe. (The ecliptic is the middle of the three rings.) On the zodiac are several familiar constellations: (1) Taurus, (2) Gemini, (3) Cancer, and (4) Leo. A number of nonzodiacal constellations may also be seen: (6) Canis Major, (7) Argo, (8) Hydra, (9) Crater, (10) Corvus, and (15) Auriga. Celestial circles represented on the Farnese globe include the celestial equator *CD*, the tropic of Capricorn *EF*, the tropic of Cancer *OK*, and the solstitial colure *AB*. (The solstitial colure is a great circle that passes through the celestial poles and the summer and winter solstitial points.)

In the placement and representation of the constellations, the globe is consistent with the descriptions of the sky in the *Phenomena* of Aratus. For example, the constellation Hercules is, in Aratus, simply called the Kneeling Man. (The identification with the hero came after Aratus's time.) Hercules is usually depicted carrying a club and a lion's skin, which is not the case with the Kneeling Man on the Farnese globe.

The Farnese globe was, of course, a display piece and not a usable globe. Figure 2.8 shows one of the oldest known portable globes, from medieval Islam.[7] Although no portable globe (of the type suitable for teaching) has come down to us from Greek times, we know that they were fairly common. Geminus, the author of an introductory astronomy textbook (*Introduction to the Phenomena*, first century A.D.), refers to celestial globes in several passages and clearly expected his readers to be familiar with them. Moreover, globes appear in Greek and Roman art, for example, on coins and on murals. (See fig. 5.13 for a coin from Roman Bithynia that shows Hipparchus seated before a small celestial globe.)

Ptolemy (*Almagest* VIII, 3) gives detailed directions for building a celestial globe. He says it is best to make the globe of a dark color, resembling the night sky, and gives directions for locating the stars on it. The stars are to be yellow, with sizes that correspond to their brightnesses. A few stars, for example, Arcturus, that appear reddish in the sky, should be painted so. The globe described by Ptolemy was of unusual sophistication, for it was fitted with a stand that allowed the user to duplicate not only the daily rotation about the poles of the equator, but also the slow precession about the poles of the ecliptic.

Similar to the celestial globe, but easier to construct, is the armillary sphere, in which the heavens are represented not by a solid ball but by a few rings or bands which form a kind of skeleton sphere. ("Armillary" from the Latin *armilla*, arm-band, bracelet.) This model emphasizes the various circles in the sky that are associated with the Sun's motion. Figure 2.9 shows a Renaissance illustration of an armillary sphere.

FIGURE 2.8. An Arabic celestial globe (Oxford, Museum of the History of Science). The stars are represented by inlaid silver disks, with sizes corresponding to the magnitudes of the stars. The globe is pierced by holes at the poles of the equator and at the poles of the ecliptic. (There is a third pair of holes whose function is not obvious. Perhaps they were drilled by mistake.) A series of holes in the stand permits adjustment of the axis of rotation for geographical latitude at increments of 10°. The inscription informs us that the globe was made in A.H. 764 (A.D. 1362/1363) and that the maker of the globe took the star positions from the *Book of the Constellations* of al-Ṣūfī. Ursa Major may be seen, upside down, near the middle of the globe. The Pointers point at the middle hole, which is the pole of the equator. (Compare with fig. 1.15.)

### *Uses of the Globe*

With either a globe or an armillary sphere it is possible to reproduce a variety of astronomical events—the risings and settings of stars, the annual solar cycle, and so on. One can make apparent in a moment what would require months to observe in the sky, so the models can be used to supplement, or even replace, real observation in the teaching of astronomy.

Even after the development of spherical trigonometry (by the end of the first century A.D.), concrete models continued to serve as aids to visualization and understanding. Indeed, if one desires only numerical answers (rather than mathematical formulas), and if one does not insist that these numbers be very precise, one can perform all the trigonometric "calculations" necessary to astronomy by manipulating a concrete model. A well-made celestial globe or armillary sphere is a kind of analog computer.[8]

These models were also aids in the discovery of the world. Many facts about the Earth are read directly on the celestial globe: the existence of a

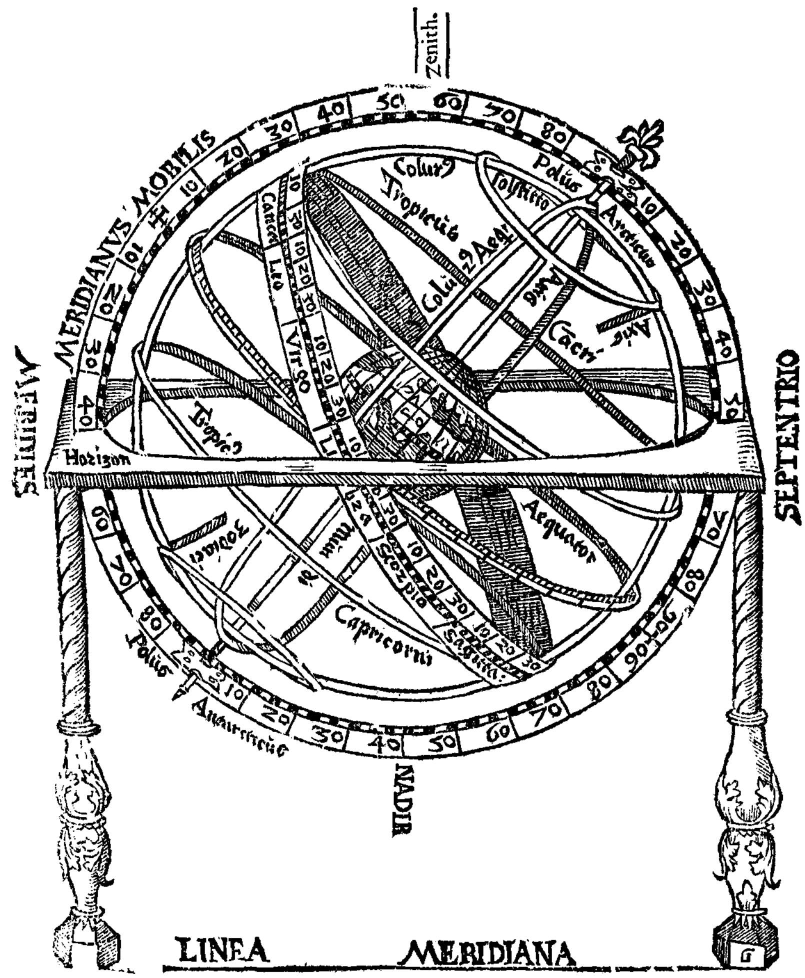



FIGURE 2.9. An armillary sphere. From *Cosmographia . . . Petri Apiani & Gemmae Frisii* (Antwerp, 1584). Courtesy of the Rare Book Collection, University of Washington Libraries.

midnight Sun in the extreme northern and southern latitudes, "days" of six months at the poles, the existence of a tropical zone in which the Sun sometimes stands at the zenith, and the reversal of the seasons in the southern hemisphere. All these facts of geography can be demonstrated with the celestial globe. These facts about the Earth were discovered through thought and not by exploration.

Armillary spheres were common teaching tools in Greek antiquity, and they are mentioned as such by Geminus. But if an armillary sphere is made well enough, and large enough, and equipped with sights, it can also function as an instrument of observation. It can be used, for example, to measure the celestial coordinates of stars or planets in the night sky. By Ptolemy's time, the armillary sphere had become the preferred instrument of the Greek astronomers. (See fig. 6.8 for the instrument described by Ptolemy in *Almagest* V, 1.)

*History of Model-Making*

According to Sulpicius Gallus,[9] Thales of Miletus (sixth century B.C.) was the first to represent the heavens with a sphere. This would have been a solid sphere on which stars were marked. Whether this sphere was made to turn about an axis we do not know. Indeed, because most of what we know of Thales is mere rumor and legend, it is far from certain that the celestial globe really originated with him. Among the ancients, Thales' name was a catchword for wisdom and learning and many discoveries were attributed to him that really were made much later.

In any case, by the time of Plato (fourth century B.C.) such models must have been fairly common. When Plato described the creation of the universe by the craftsman-god in his *Timaeus*, he had in mind the physical image of the universe provided by the armillary sphere. According to Plato, the craftsman-god first of all prepared a fabric from which he intended to construct the world, and this fabric was made of world-soul. The craftsman-god

> then took the whole fabric and cut it down the middle into two strips, which he placed crosswise at their middle points to form a shape like the letter X; he then bent the ends round in a circle and fastened them to each other opposite the point at which the strips crossed, to make two circles, one inner and one outer. And he endowed them with uniform motion in the same place, and named the movement of the outer circle after the nature of the Same, of the inner after the nature of the Different. The circle of the Same he caused to revolve from left to right, and the circle of the Different from right to left on an axis inclined to it; and he made the master revolution that of the Same.[10]

"Motion in the same place" means circular motion. The two intersecting circles are, of course, the equator and the ecliptic. The daily motion from east to west, shared by all the heavenly bodies, is the "master revolution," or the revolution "of the Same," and is associated with the equator. The ecliptic partakes of the nature of the Different because the Sun, Moon, and planets all tend to move in the contrary direction—from west to east—along this circle. There is no doubt, then, that Plato's conception of the universe owed something to the concrete example of the armillary sphere. This is perhaps the earliest example we have of something that has since become commonplace: a successful scientific model or theory may affect our picture of the world and cause shifts in religion and philosophy.

Farther on in the same discussion, Plato mentions the creation of the planets and the motions with which god has endowed them. But he forswears any detailed explanation of these motions, saying, "It would be useless without a visible model to talk about the figures of the dance [of the planets]," which again makes one think that models were in use by Plato's time.

Eudoxus of Cnidus sought to explain this dance of the planets by a system of nested spheres, turning about several different axes inclined to one another. He was able in this way to reproduce fairly well the variations in speed, the stationary points, and the retrogradations that are characteristic of the planets' motions. Whether he made a concrete model to illustrate his theory is not known. Eudoxus, who was a mathematician of the first order, would not have needed mechanical aids, but such a model might have made discussion with others easier. If it existed, the model of Eudoxus would have been the first orrery. Such a device, which duplicates the motions of the planets, is much more complicated than a globe or armillary sphere, which merely reproduces the daily revolution of the celestial sphere.

In any case, *sphairopoiïa* ("sphere making")—the art of making models to represent the celestial bodies and their motions—soon became an established branch of mechanics and was carried to a high level by the time of Archimedes (ca. 250 B.C.). According to Plutarch,[11] this brilliant mathematician repudiated as sordid and ignoble the whole trade of mechanics and every art that lent itself to mere use and profit. Archimedes is famous for inventing machines of all kinds—water screws, hoisting machines, and engines of war—but these he is supposed to have designed not as matters of any importance but as mere amusements in geometry. And so Archimedes did not "deign to leave behind him any commentary or writing on such subjects" but "placed his whole affection and ambition in those purer speculations where there can be no reference to the vulgar needs of life." Yet, he seems to have made an exception in the case of sphere making, perhaps because it helps one attain

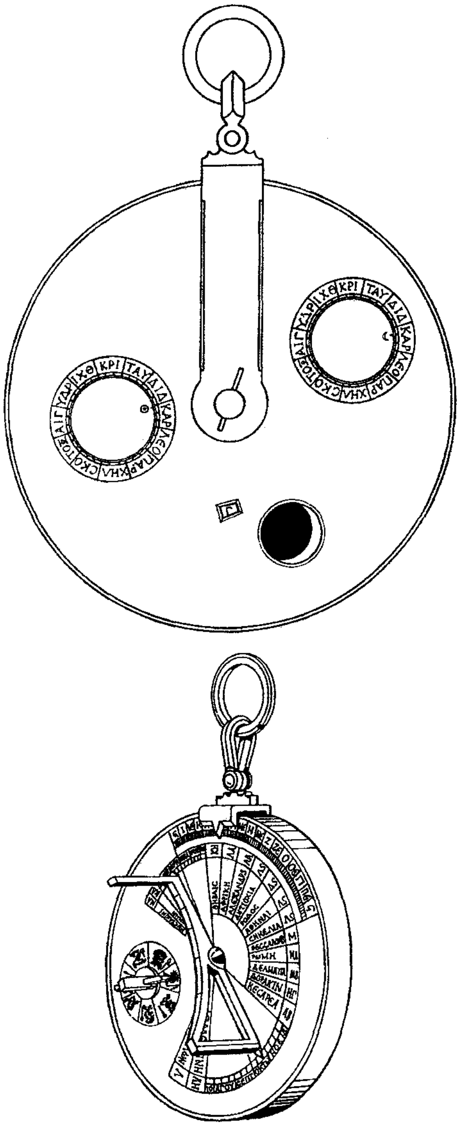


FIGURE 2.10. A. A portable sundial and gear-work calendrical calculator from the Byzantine period (circa A.D. 500). *Top*: Conjectural reconstruction of the back. A dial at the left indicates the position of the Sun in the zodiac. A dial at the right indicates the position of the Moon. The window at the bottom indicates the phase of the Moon. *Bottom*: Reconstruction of the front. The suspension ring must be positioned for the latitude of the observer. The sundial vane must be adjusted for the time of year. The user holds the dial vertically and turns it until the shadow of the gnomon falls on the scale of hours engraved on the curved part of the vane, indicating the time of day. The dial at the left is to be turned one notch a day. The gear train inside the device then advances the Sun and Moon indicators by the appropriate amounts. From Field and Wright (1984).

an understanding of divine objects. For, according to Pappus,[12] Archimedes composed a special treatise on this subject, which is now lost. This book on sphere making was the only work on mechanics that Archimedes judged worthwhile to write.

That Archimedes actually made models of the heavens is beyond doubt, for Cicero tells us that after the capture of Syracuse (212 B.C.) the Roman general Marcellus brought two of them back to Rome. One was a solid celestial globe, which Marcellus placed in the temple of Vesta, where all might go and see it. This seems to be the same globe that Ovid mentions in these lines on Vesta and her hall:

> There stands a globe hung by Syracusan art
> In closed air, a small image of the vast vault of heaven,
> And the Earth is equally distant from the top and bottom.
> That is brought about by its round shape.[13]

Ovid's description of the sphere as an image of the heavens with the Earth inside, equally distant from top and bottom, makes it sound more like a hollow armillary sphere than the solid globe described by Cicero. Ovid wrote these lines around A.D. 8, more than 200 years after the globe was brought to Rome. And, although Ovid writes as if the globe still existed in his time, it is possible that it did not and that he never saw it. Besides, Ovid's astronomical knowledge is often defective, so Cicero's description is more to be trusted.[14]

According to Cicero, the second of Archimedes' models was taken home by Marcellus, "though he took home with him nothing else out of the great store of booty captured." Years later, it was shown by Marcellus's grandson to Gaius Sulpicius Gallus, who was evidently one of the few who understood the workings of the machine. This second model,

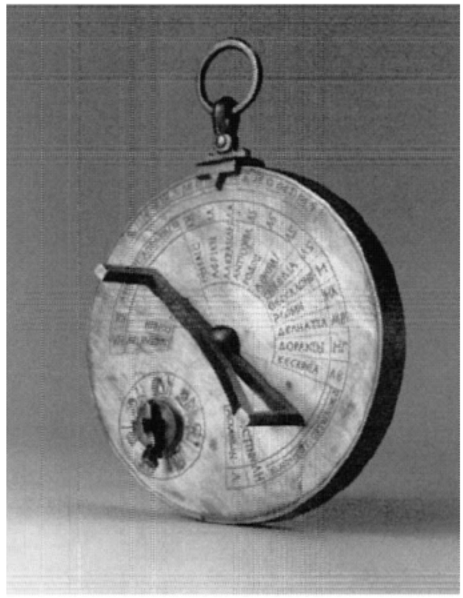
> on which were delineated the motions of the Sun and the Moon and of those five stars which are called wanderers, or as we might say, rovers, contained more than could be shown on the solid globe, and the invention of Archimedes deserved special admiration because he had thought out a way to represent by a single device for turning the globe those various and divergent movements with their different rates of speed. And when Gallus moved the globe, it was actually true that the Moon was always as many revolutions behind the Sun on the bronze contrivance as would agree with the number of days it was behind in the sky. Thus, the same eclipse of the Sun happened on the globe as would actually happen. . . .[15]

This orrery of Archimedes must have been quite a marvel, for Cicero expresses disapproval of some who "think more highly of the achievement of Archimedes in making a model of the revolutions of the firmament than that of nature in creating them, although the perfection of the original shows a craftsmanship many times as great as does the counterfeit."[16]

Archimedes was not the only master of the art of sphere making, for Cicero also mentions "the orrery recently constructed by our friend Posidonius, which at each revolution reproduces the same motion of the Sun, the Moon and the five planets that take place in the heavens every twenty-four hours." Cicero probably saw this device himself, for as a young man he had attended Posidonius's lectures in Rhodes, and again befriended him when the philosopher came to Rome as ambassador from Rhodes in 87–86 B.C. But, alas, Cicero gives us no details of the construction of this machine.

The orreries of Archimedes and Posidonius were intended primarily to give a visual representation of the universe. But it is clear from Cicero's remarks that these two orreries also incorporated some quantitative features of the planets' motions—at least their relative speeds along the zodiac. Two related kinds of constructions can be mentioned here. One was the simple cosmological model, which did not incorporate any quantitative features, but which gave the viewer an overall visual impression of the arrangement of the universe. For example, Theon of Smyrna[17] tells us that he himself made a model of the nested spindle-whorls cosmos described by Plato in the tenth book of the *Republic*. This model would not have *done* much, but it did illustrate Plato's cosmology in a visually striking way.

Less visual, but more quantitative, was the gearwork calendrical computer. Parts of two such devices have been discovered, one dating from the first century B.C.[18] and one from the fifth or sixth century A.D.[19] The user was expected to turn a wheel through one "click" each day. A gearwork mechanism then advanced indicators showing the phase of the Moon and the position of the Sun in the zodiac (see fig. 2.10).



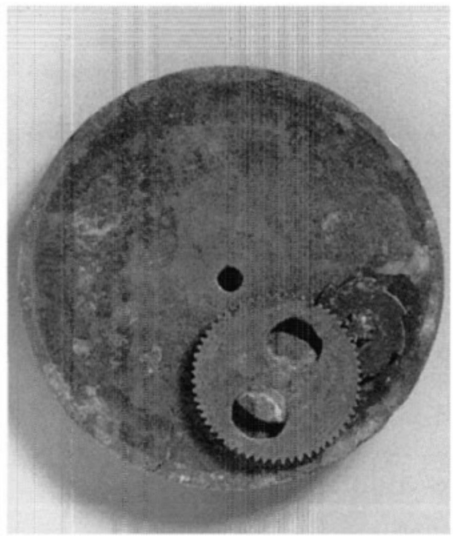


FIGURE 2.10. B. Portable sundial and gearwork calendrical calculator from the Byzantine period. *Top:* A modern reconstruction in metal. *Bottom:* The extant portion of the gear train. At the right can be seen the ratchet (the oldest known ratchet), which prevented the user from turning the day dial in the wrong direction. Science Museum, London.

### *The Place of Sphairopoiïa among the Mathematical Arts*

One should not take Archimedes' disdain for mechanics as representative of his time. By Archimedes' time, mechanics not only was a useful trade, but also had become a recognized genre of technical writing. *Sphairopoiïa*, the subdivision of mechanics devoted to models of the heavens, was also a recognized specialty. Sphairopoiïa included the construction of celestial globes, to be sure. But, as we have seen, it also included the making of other kinds of images of the heavens, such as models of the planetary system and mechanical calculating devices intended to replicate features of the motions of the Sun, Moon, and planets. Two recognized branches of astronomy proper were also devoted to concrete constructions: *gnomonics* (the making of sundials) and *dioptrics* (the design and use of sighting instruments).

The relation of these three arts to the rest of mathematical knowledge is discussed by Geminus, a Greek scientific writer of the first century A.D. Geminus wrote an elementary astronomy textbook (*Introduction to the Phenomena*) that has come down to us more or less intact. He also wrote a large book on mathematics, which contained a good deal of philosophy and history of mathematics. This book has not come down to us. But much of its content is summarized by Proclus in his *Commentary on the First Book of Euclid's Elements*. In his mathematical treatise, Geminus discussed the organization of mathematical knowledge and the relation of its various branches to one another. Geminus's outline of the mathematical sciences can be summarized thus:

Organization of the mathematical sciences according to Geminus

- Pure mathematics (concerned with mental objects only)
  - Arithmetic (study of odds, evens, primes, squares)
  - Geometry
    - Plane geometry
    - Solid geometry
- Applied mathematics (concerned with perceptible things)
  - Practical calculation (analogous to arithmetic)
  - Geodesy (analogous to geometry)
  - Theory of musical harmony (an offspring of arithmetic)
  - Optics (an offspring of geometry)
    - Optics proper (straight rays, shadows, etc.)
    - Catoptrics (theory of mirrors, etc.)
    - Scenography (perspective)
  - Mechanics
    - Military engineering
    - Wonderworking (pneumatics applied to automata)
    - Equilibrium and centers of gravity
    - Sphere making (mechanical images of the heavens)

FIGURE 2.11. A sixteenth-century brass armillary sphere. Science Museum, London.

- Astronomy
  - Gnomonics (sundials)
  - Meteoroscopy (general astronomical theory)
  - Dioptrics (instruments of observation)[20]

Mathematical knowledge is divided into the pure (which deals with mental objects only) and the applied (which deals with perceptible things). Astronomy is one among six branches of applied mathematics. Geminus was not alone in making astronomy a part of mathematics, for this was the general view among the Greeks. Ptolemy, for example, always refers to himself as a *mathematician*. Two of Geminus's three subbranches of astronomy are concerned with the construction and use of instruments: *gnomonics* and *dioptrics*. As Geminus says, gnomonics is concerned with the measurement of time by means of sundials, while "dioptrics examines the positions of the Sun, Moon, and the other stars by means of just such instruments [i.e., dioptras]." As for sphere making, Geminus makes it a part of mechanics, no doubt because it involves the use of geared mechanisms and water power to activate its images of the heavens.

### *Some Reservations about Sphairopoiïa*

The purpose of sphairopoiïa was to make immediately evident facts that could otherwise demonstrated only by difficult geometrical argument or prolonged observation of the skies. The danger of this method was that the desire to perfect the concrete mechanism would replace the taste for reflection and observation and so would lead one away from real astronomy into simple tinkering. Plato had already criticized the geometers who made use of mechanical devices to solve problems,[21] saying that this was the corruption and annihilation of the one good of geometry, which ought to concern itself with the contemplation of the unembodied objects of pure intelligence, rather than with base material things. And Ptolemy, the greatest astronomer of antiquity, objected to traditional sphere making on the grounds that, in the majority of cases, it only reproduced the appearances of things without troubling itself over causes and gave proofs of its own technical accomplishment rather than of the justice of astronomical hypotheses.[22] Ptolemy's complaint probably was justified, especially when applied to the orreries, which certainly had an air of the marvelous and extravagant. But there is no doubt that the simpler models—the armillary sphere and the celestial globe—played an important part in the teaching of astronomy and even, in the early days of this science, in fundamental research and discovery. Perhaps it was because he realized this that Ptolemy, in his *Planetary Hypotheses*, decided after all to give a summary of ideas that might be useful to those who wish to make concrete models of the cosmos.

### *A Renaissance Armillary Sphere*

In the Renaissance, armillary spheres became enormously popular, and many examples survive in museums.[23] In figure 2.11 we see a well-made, functional model, suitable for instructional use. This armillary sphere is of sixteenth-century German workmanship. The circles are of brass. The outside diameter of the meridian is about 9 1/2 inches. This sphere has an interesting special feature: it is equipped with rotatable auxiliary rings that allow markers representing the Sun and Moon to be moved and positioned at will along the zodiac. The Sun and Moon markers may be seen on the inside of the zodiac ring.

## 2.3 EXERCISE: USING A CELESTIAL GLOBE

### *Directions for Use of the Sphere*

A usable celestial globe must have the following features: (1) a fixed horizon stand, (2) a moveable meridian ring that allows the model to be adjusted for the observer's latitude, and (3) an axis of rotation. (These features are all displayed by the Renaissance model in fig. 2.9.) If you use a solid celestial globe, you should visualize the Earth as a geometric point, at rest at the center of the sphere.

The four most important circles of the model are the horizon, the meridian, the equator, and the ecliptic.

The horizon and the meridian are fixed circles that do not participate in the revolution but form a base or stand for the revolving sphere. The *horizon ring* represents the observer's own horizon. Therefore, points of the sphere that are above the horizon are visible, and those below, invisible. The horizon should be marked all around at 5° or 10° intervals. On most horizon stands, the cardinal points (north, east, south, and west) also are marked. These markings enable one to tell in just what direction a given star rises or sets.

The *meridian ring* may be turned in the stand so that the elevation of the celestial pole above the horizon may be varied. By this means the model may be adjusted to give the appearance of the sky at any desired latitude. The latitude of a place on Earth is equal to the altitude of the north celestial pole (or arctic pole) at that place.

The equator and the ecliptic both participate in the daily revolution of the heavens. The *equator* is divided into hours. These marks may be counted as they turn past the meridian ring to measure elapsed time. In other words, the celestial equator, turning past the fixed meridian, constitutes a giant clock. Thus, one may determine, for example, the time between the rising and setting of a particular star. (Technically, the stars take about four minutes less than twenty-four hours to complete a revolution. For most purposes this small difference may be ignored.)

The *ecliptic* is the path that the Sun follows in its annual motion. On your model, it may be marked in degrees of celestial longitude, or with the dates on which the Sun reaches each point, or with both kinds of information.

If you are using a celestial globe, you will see that it is marked with many stars. If you are using an armillary sphere, it may be marked with the approximate positions of a few prominent stars that happen to lie on or near one of the circles.

### *Example*

*Problem*: What will an observer at 50° north latitude see the Sun do on April 20?

*Solution*: First set the meridian so that the arctic pole is 50° above the north point of the horizon (as in fig. 2.9).

Then place the April 20 mark of the ecliptic on the horizon and find that the Sun rises about 17° north of east. Note that the 19-hour mark of the equator is now at the meridian. (This is, in modern parlance, called the *sidereal time* of sunrise. The sidereal time is indicated by the hour mark of the equator that is on the meridian above the horizon. Sidereal time is *not* the same as ordinary clock time.)

Then turn the sphere until the April 20 mark reaches the western horizon and note that the 8 3/4 hour mark of the equator is on the meridian. (Thus, the sidereal time of sunset is 8 3/4 hours.) To find the length of the day, subtract the sidereal time of sunrise from the sidereal time of sunset: the Sun was above the horizon for 8 3/4 − 19 = 24 + 8 3/4 − 19 = 13 3/4 hours.

A note on reckoning time intervals: In computing a time interval, the rule is always (time of final event) – (time of initial event). So, to find the length of the day, we compute sunset minus sunrise. If you subtract in the wrong order, you will find the length of the night. If you cannot carry out the subtraction because the first number is smaller than the number being subtracted, you can always add 24 hours to the first number, as in ordinary clock arithmetic.

Finally, place the Sun (the April 20 mark) at the meridian, simulating local noon. Find that the Sun is 51° above the horizon.

In summary, on April 20, an observer at 50° north latitude will see the Sun rise 17° north of east, cross the meridian 51° above the horizon, and set 17° north of west, 13 3/4 hours after it rose.

### *The Motion of the Sun*

Use a celestial globe or an armillary sphere to investigate the behavior of the Sun at different times of the year and at different latitudes. In particular:

1. Make a graph of the altitude of the Sun at noon versus the date. Plot at least one point for each thirty days over an entire year. You should make three such graphs, for latitudes 0°, 35°, and 70°. Your graphs will be more meaningful if you display them on a single sheet of graph paper.
2. Make a graph of the rising direction (number of degrees north or south of east) of the Sun versus the date for a whole year. Do this for each of the same three latitudes.
3. Make a graph of the length of the longest day of the year (in hours) versus latitude. Vary the latitude by 10° steps from 0° to 90°. (This graph has a historical as well as an astronomical interest. The Greeks often designated the latitude of a place by giving the length of the longest day there.)

### *Questions and Problems*

1. Is there any place on Earth at which the Sun rises directly in the east every day of the year?
2. Is there any time of year at which the Sun rises directly in the east everywhere on Earth?
3. Use the celestial globe to determine the truth or falsity of the following two familiar statements: "At the equator, the Sun always rises directly in the east. Moreover, the Sun is above the horizon twelve hours every day there."
4. Suppose the Sun crosses the local meridian south of the zenith at some particular place on Earth and on some particular day. Can there be any place on Earth at which the Sun crosses the meridian north of the zenith on that same day?
5. Suppose the Sun rises south of east at some particular place on Earth and on some particular day. Can there be any place on Earth at which the Sun rises north of east on that same day?
6. The tropic of Cancer that is often marked on globes of the Earth is a projection of the celestial tropic of Cancer. Therefore, it is a circle on the Earth's surface at a latitude of about 23°. What is special about this latitude? In what way are latitudes above this different from those below? Think in terms of the apparent motion of the Sun.
7. The arctic circle is a circle on the Earth at a latitude of about 67°. In what way are latitudes above the arctic circle different from those below?
8. Suppose we divide the Earth into five zones, with boundaries formed

by the arctic circle, the tropic of Cancer, the tropic of Capricorn, and the antarctic circle. Describe characteristics of each zone as fully as possible, in terms of the Sun's behavior.

The Greeks divided the Earth into these same zones, but some writers limited the frigid zones by the arctic and antarctic circles of the Greek horizon. See section 2.5 for an explanation of the "local arctic circle." On the zones, see section 1.12.

## 2.4 EARLY WRITERS ON THE SPHERE

*Autolycus of Pitane*

The oldest surviving works of Greek mathematical astronomy are those of Autolycus, *On the Moving Sphere* and the two books called *On Risings and Settings*.[24] Autolycus (roughly 360–290 B.C.) came from the city of Pitane on the western coast of Asia Minor, opposite Mytilene. His works date from the time when Greek mathematical astronomy was just emerging. Together, the three works contain several dozen propositions, all simply and geometrically proved.

*On the Moving Sphere* treats twelve elementary propositions concerning a sphere that rotates about a diameter as axis. For example,

1. If a sphere rotates uniformly about its axis, all the points on the surface of the sphere which are not on the axis will trace parallel circles that have the same poles as the sphere, and that are perpendicular to the axis.

One notices here something that is common in all the elementary astronomical works: a reversal of the line of historical development. Thus, although astronomy began with observation of the circular motion of the stars, from which the spherical form of the heavens was inferred, Autolycus *assumes* a spherical universe and deduces the circular orbits of the stars.

4. If on a sphere an immobile great circle perpendicular to the axis separates the invisible from the visible hemisphere, then during the rotation of the sphere about its axis, none of the points on the surface of the sphere will set or rise. Rather, the points located on the visible hemisphere are always visible; and those on the invisible hemisphere are always invisible.

The "immobile great circle that separates the invisible from the visible hemisphere" is the horizon. Circumlocutions such as this were common in an age in which a technical vocabulary was still emerging. Our term *horizon* derives from the Greek verb *horizo*, to divide or separate. In this fourth proposition, then, Autolycus considers a situation in which the axis of the universe is perpendicular to the horizon. Such is the case at the north or south pole of the Earth, where the celestial pole stands directly overhead. Here, none of the stars rise or set.

A curious aspect of Autolycus's style in *On the Moving Sphere* is the absence of any overt reference to the astronomical applications of the theorems. The objects that rise and set are not stars but merely points (*semeia*), and the object on which these points are fixed is not the cosmos but a hypothetical revolving sphere. This was probably deemed to make the book better (because purer) geometry.

5. If a fixed circle passing through the poles of the sphere separates the visible from the invisible part, all points on the surface of the sphere will, in the course of its revolution, both set and rise. Further, they will pass the same time below the horizon and above the horizon.

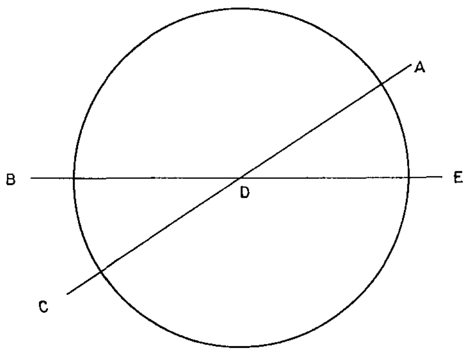



FIGURE 2.12.

Proposition 5 applies at the Earth's equator, where the celestial poles lie on the horizon.

9. If on a sphere a great circle oblique to the axis separates the visible part of the sphere from the invisible part, then, of all the points that rise at the same time, the ones closer to the visible pole will set later; and, of all the points that set at the same time, the ones closer to the visible pole will rise sooner.

If the horizon is neither perpendicular nor parallel to the axis, it is said to be oblique or inclined. So proposition 9 applies wherever 4 and 5 do not—that is, everywhere on Earth except at the poles and the equator.

The propositions quoted here give a fair idea of the subject matter and the level of difficulty of *On the Moving Sphere.* The propositions are all of the kind that could be discovered by experiment on a concrete model and then proved by elementary geometrical reasoning. In Autolycus we find nothing that could serve as a basis for a general method of calculation—there is as yet no trigonometry—but only knowledge of the sort that implies a thorough familiarity with the celestial globe.

### *Euclid's Phenomena*

Almost contemporary with Autolycus was Euclid, whose masterwork on geometry dates from the third century B.C. The thirteen books of the *Elements* represent the culmination of classical geometry and remain among the most studied works in the history of thought. More than a 1,000 editions have appeared since the invention of printing.[25] However, the *Elements* contains little that is of special interest for astronomy. The geometry of the sphere, for example, is scarcely treated.

But Euclid did leave us a more astronomical work, the *Phenomena,* which covers in eighteen theorems some important, if elementary, features of spherical astronomy. Let us examine a few:[26]

1. The Earth is in the middle of the cosmos and occupies the position of center with respect to the cosmos.

How does Euclid prove the centrality of the Earth? Suppose, as in figure 2.12, that *ABC* is the circle of the horizon, with *C* in the eastern part and *A* in the west. The observer is at *D.* Look through a sighting tube (dioptra) at the Crab when it is rising at *C.* If you then turn around and look through the other end of the tube you will see the Goat-Horn setting at *A.* Thus, *ADC* is a diameter of the sphere of stars, for the arc between the Goat-Horn and the Crab is six zodiac signs. In the same way, aim the dioptra at *B* when the Lion is rising there. If you then look through the other end of the dioptra, you will see the Water-Pourer setting at *E. B* and *E* are six signs apart, so *BDE* is also a diameter of the sphere of stars. Therefore, the point *D* of intersection is the center of the sphere. This is the Earth, where the observer stands.

Like Ptolemy's demonstrations of the place of the Earth, Euclid's argument is a thought-proof rather than a true appeal to observation. No one ever became convinced of the centrality of the Earth by making such observations. The conventional nature of the proof is clear from Euclid's use of Crab and Goat-Horn as if they were points on the sphere rather than zodiacal signs each 30° long. Conventional demonstrations can have a long life. Copernicus, for example, some 1,800 years later, gave exactly the same "proof" that the Earth is as a point in comparison with the heavens.[27] Even the figure is the same. Copernicus, of course, deduces from these considerations only the smallness of the Earth: he points out, rightly, that this evidence does not prove that the Earth is at the center of the universe.

3. Of the fixed stars that rise and set, each [always] rises and sets at the same points of the horizon.

Euclid proves this from the spherical nature of the heavens, but this elementary fact was certainly known long before anyone had any conception of the celestial sphere.

11. Of [two] equal and opposite arcs of the ecliptic, while the one rises the other sets, and while the one sets the other rises.

17. Of [two] equal arcs [of the ecliptic] on either side of the equator and equidistant from the equator, in the time in which one passes across the visible hemisphere the other [passes across] the invisible hemisphere. . . .

These theorems, like those in Autolycus, contain little that would be useful to a practicing observer, nor are they the results of observation. Rather, they are simple consequences of the spherical nature of the heavens, sprung from the realm of thought.

### *The* Little Astronomy

Euclid's *Phenomena* and Autolycus's *On the Moving Sphere* are preserved in Greek manuscripts of the medieval period—manuscripts that were copied by hand more than a thousand years after the originals were set down by their authors. In many cases, these two works are found bound together with a number of other minor works of Greek astronomy. As an example, take the manuscript *Vaticanus graecus* 204 (i.e., Greek manuscript no. 204 in the Vatican Library). This manuscript is valuable for our knowledge of Autolycus, both because of the care with which it was copied, and because of its age: it dates from the ninth or tenth century A.D., which makes it the oldest surviving copy of Autolycus's Greek text. The partial contents of this manuscript are as follows:[28]

- Theodosius of Bithynia, *Spherics*. First century B.C. The *Spherics* is a treatise on the geometry of the sphere, in the style of Euclid's *Elements*. The *Spherics* of Theodosius may be considered a continuation of, and a supplement to the *Elements*. Thus, it is more sophisticated than the *Phenomena* of Euclid.
- Autolycus, *On the Moving Sphere*.
- Euclid, *Optics*. This is an elementary geometrical treatise on various effects involving the straight-line propagation of light: shadows, perspective, parallax, and so on. Example: When one observes a sphere with both eyes, if the diameter of the sphere is equal to the distance between the pupils, one will see exactly half the sphere; if the distance between the pupils is greater, one will see more than half; if the distance between the pupils is less, one will see less than half. Second example: If several objects move at the same speed, the most distant will appear to move most slowly.
- Euclid, *Phenomena*.
- Theodosius of Bithynia, *On Geographic Places*. This little book, similar in flavor to Euclid's *Phenomena* and Autolycus's *On the Moving Sphere*, describes, in twelve propositions, the appearance of the sky as seen from various places on the Earth. Example: an inhabitant of the north pole would see always the northern hemisphere of the celestial sphere; the southern hemisphere would be forever unseen; no star would rise or set.
- Theodosius of Bithynia, *On Days and Nights*. This work presents thirty-one propositions concerning the lengths of the days and nights at different times of the year, at different latitudes on the Earth.
- Aristarchus of Samos, *The Sizes and Distances of the Sun and Moon*, third century B.C. (discussed in sec. 1.17).

- Autolycus, *On Risings and Settings* (discussed in sec. 4.9).
- Hypsicles, *On Ascensions*, second century B.C. In this short treatise, Hypsicles proves a number of propositions on arithmetical progressions and uses the results to calculate approximate values for the times required for the signs of the zodiac to rise above the horizon. Such information had practical applications, for example, in telling time at night. Hypsicles' treatise is discussed in section 2.16.
- Euclid, *Catoptrics*. This work, whose attribution to Euclid is disputed, is concerned with what we would today call optics proper. It treats the reflection of light and the formation of images by mirrors.
- Euclid, *Data* (= "given"). This treatise on elementary geometry consists of propositions proving that, if certain things in a figure are given, something else may be figured out.

This hodgepodge of short, elementary astronomical and geometrical works is found in many medieval manuscripts. Sometimes one or more are wanting. Often, other minor works are present, such as commentaries on the mathematical works of Apollonius and Euclid. The particular collection of short works listed above is sometimes called the *Little Astronomy*. Also usually included is the *Spherics* of Menelaus (first century A.D.), which no longer survives in Greek but is known through Arabic translations. Menelaus's work treats the geometry of spherical triangles. It has been said that, from the second century A.D. onward, the *Little Astronomy* served as an introductory-level textbook for students who were not yet prepared to tackle the "Big Astronomy," that is, the *Almagest* of Ptolemy.[29] It may have been so, but the evidence is slight.[30] Indeed, the chief evidence is simply the fact that many Byzantine manuscripts contain more or less the same assortment of elementary astronomical and geometrical works.

The supposed title of the collection is provided by a remark at the beginning of the sixth book of the *Mathematical Collection* of Pappus of Alexandria. Of Pappus himself we know very little. He lived and taught at Alexandria, during the last half of the third and the first half of the fourth century A.D. He had a son, Hermodoros, to whom he addressed two of his books. He had as friends two geometers, Pandrosios and Megethios, who are otherwise unknown. Pappus wrote a commentary on the *Almagest* of Ptolemy, which survives in part. But his most important work is the one that has come down to us under the title *The Mathematical Collection of Pappus of Alexandria*. This consists of a vast collection of propositions extracted from a great number of works on mathematics, astronomy, and mechanics (many of which are lost today), accompanied by Pappus's explanatory notes, alternative demonstrations, and new applications. The work does not seem to have been written according to any plan, but was probably the result of many years' reading and note taking, no doubt in connection with Pappus's teaching duties at Alexandria. The sixth book of the collection is devoted to the astronomical writers. Pappus discusses works by Theodosius, Menelaus, Aristarchus, Euclid, and Autolycus. At the beginning of the sixth book, we find the remark, written as a subtitle, "It contains the resolutions of difficulties found in the little astronomy."

Whether or not there really existed a definite collection of treatises known as the *Little Astronomy*, there is no doubt that the individual works were used by teachers from the late Hellenistic period down to Byzantine times. The tradition was continued by Arabic teachers, who made use of the same treatises in translation and added others as well. It was the schoolroom usefulness of these works that guaranteed their survival, for many works of greater scientific and historical importance have been lost, for example, most of the writings of Hipparchus and all those of Eudoxus.

Aristarchus's work, *On the Sizes and Distances of the Sun and Moon*, is quite different from the others of the collection: Aristarchus attempted to arrive at new astronomical knowledge by calculations based on astronomical data. The rest of the purely astronomical works of the *Little Astronomy* are theoretical developments of various properties of the celestial sphere, devoid of any reference to particular observations. The oldest works of the collection, those of Autolycus and Euclid (ca. 300 B.C.), represent the first attempts to grapple with the problems of spherical geometry, and therefore are endowed with a great historical interest. Some of the later works, for example, Theodosius's treatises *On Geographical Places* and *On Days and Nights* (ca. 100 A.D.), lag considerably behind the astronomical and mathematical knowledge of their own day and must actually have been written as primers for students. Their elementary nature and pedantic style would reveal them as textbooks in any age. Taken together, the treatises of the *Little Astronomy* illustrate the level of Greek mathematical astronomy around the beginning of the second century B.C., before the revolution in calculating ability brought about by the development of trigonometry. Menelaus's book was one that helped point the way to the new mathematics.

## 2.5 GEMINUS: *INTRODUCTION TO THE PHENOMENA*

In addition to the works of the *Little Astronomy*, we have several other elementary texts from a slightly later period. A notable example is the *Introduction to the Phenomena* by Geminus, a writer of the first century A.D.[31] This work is sometimes called the *Isagoge*, from the first word of its Greek title. This work differs markedly from most of those in the *Little Astronomy*. In the first place, it is longer. And second, it is written with grace and style. It is, in fact, a well-organized and more or less complete introduction to astronomy, intended for beginning students of this subject.

Geminus takes up the zodiac and the motion of the Sun, the constellations, the celestial sphere, days and nights, the risings and settings of the zodiacal signs, luni-solar periods and their application to calendars, phases of the Moon, eclipses, star phases, terrestrial zones and geographical places, and the foolishness of making weather predictions by the stars. From this lively and readable book we have extracted some sections devoted to the principal circles of the celestial sphere.

Italicized subheadings in the extract do not appear in the original, but have been added for the reader's convenience. Likewise, the numbering of statements is not a part of the original text, but is a practice introduced by modern scholars for their own convenience. An asterisk (*) in the text indicates that an explanatory note, keyed to the statement number, follows the extract.

### EXTRACT FROM GEMINUS

*Introduction to the Phenomena* V

*The Circles on the Sphere*

*1* Of the circles on the sphere, some are parallel, some are oblique, and some [pass] through the poles.

*The Parallel Circles*

The parallel [circles] are those that have the same poles as the cosmos. There are 5 parallel circles: arctic [circle], summer tropic, equinoctial,* winter tropic, and antarctic [circle].

*2* The arctic circle* is the largest of the always-visible circles, [the circle] touching the horizon at one point and situated wholly above the Earth. The stars lying within it neither rise nor set, but are seen through the whole night turning around the pole. *3* In our *oikumene*,* this circle is traced out by the forefoot of the Great Bear.*

*4* The summer tropic circle is the most northern of the circles described by the Sun during the rotation of the cosmos. When the Sun is on this circle, it produces the summer solstice, on which occurs the longest of all the days of the year, and the shortest night. *5* After the summer solstice, however, the Sun is no longer seen going towards the north, but it turns towards the other parts of the cosmos, which is why [this circle] is called "tropic."*

*6* The equinoctial circle is the largest of the 5 parallel circles. It is bisected by the horizon so that a semicircle is situated above the Earth, and a semicircle below the horizon. When the Sun is on this circle, it produces the equinoxes, that is, the spring equinox and the fall equinox.

*7* The winter tropic circle is the southernmost of the circles described by the Sun during the rotation of the cosmos. When the Sun is on this circle it produces the winter solstice, on which occurs the longest of all the nights of the year, and the shortest day. *8* After the winter solstice, however, the Sun is no longer seen going towards the south, but it turns toward the other parts of the cosmos, for which reason this [circle] too is called "tropic."

*9* The antarctic circle is equal [in size] and parallel to the arctic circle, being tangent to the horizon at one point and situated wholly beneath the Earth. The stars lying within it are forever invisible to us.

*10* Of the 5 forementioned circles the equinoctial is the largest, the tropics are next in size and—for our region—the arctic circles are the smallest. *11* One must think of these circles as without thickness, perceivable [only] with the aid of reason, and delineated by the positions of the stars, by observations made with the dioptra, and by our own power of thought. For the only circle visible in the cosmos is the Milky Way; the rest are perceivable through reason. . . .

. . .

*Properties of the Parallel Circles*

*18* Of the 5 forementioned parallel circles, the arctic circle is situated entirely above the Earth.

*19* The summer tropic circle is cut by the horizon into two unequal parts: the larger part is situated above the Earth, the smaller part below the Earth. *20* But the summer tropic circle is not cut by the horizon in the same way for every land and city: rather, because of the variations in latitude, the difference between the parts is different. *21* For those who live farther north than we do, it happens that the summer <tropic> is cut by the horizon into parts that are more unequal; and the limit is a certain place where the whole summer tropic circle is above the Earth. *22* But for those who live farther south than we do, the summer tropic circle is cut by the horizon into parts more and more equal; and the limit is a certain place, lying to the south of us, where the summer tropic circle is bisected by the horizon.

*23* <For the horizon in Greece, the summer tropic> is cut <by the horizon> in such a way that, if the whole circle is [considered as] divided into 8 parts, 5 parts are situated above the Earth, and 3 below the Earth. *24* And it was for this clime* that Aratus seems in fact to have composed his treatise, the *Phenomena*; for, while discussing the summer tropic circle, he says:

*If it is measured out, as well as possible, into eight parts,*
*five turn in the open air above the Earth,*
*and three beneath; on it is the summer solstice.*

From this division it follows that the longest day is 15 equinoctial hours* and the night is 9 equinoctial hours.

*25* For the horizon at Rhodes, the summer tropic circle is cut by the horizon in such a way that, if the whole circle is divided into 48 parts, 29 parts are situated above the horizon, and 19 below the Earth. From this division it follows that the longest day in Rhodes is 14 1/2 equinoctial hours and the night is 9 1/2 equinoctial hours.

*26* The equinoctial circle, for the whole oikumene, is bisected by the horizon, so that a semicircle is situated above the Earth, and a semicircle below the Earth. For this reason, the equinoxes are on this circle.

*27* The winter tropic circle is cut by the horizon in such a way that the smaller part is above the Earth, the larger below the Earth. The inequality of the parts has the same variation in all the climes as was the case with the summer tropic circle, because the opposite parts of the tropic circles are always equal to one another. For this reason, the longest day is equal to the longest night, and the shortest day is equal to the shortest night.

*28* The antarctic circle is hidden wholly beneath the horizon. . . .

. . .

*45* The distances of the circles from one another do not remain the same for the whole oikumene. But in the engraving of the spheres, one makes the division in declination in the following way. *46* The entire meridian circle being divided into 60 parts, the arctic [circle] is inscribed 6 sixtieths from the pole; the summer tropic is drawn 5 sixtieths from the arctic [circle]; the equinoctial 4 sixtieths from each of the tropics; the winter tropic circle 5 sixtieths from the antarctic; and the antarctic [circle] 6 sixtieths from the pole.

*47* The circles do not have the same separations from one another for every land and city. The tropic circles do maintain the same separation from the equinoctial at every latitude, but the tropic circles do not keep the same separation from the arctic [circles] for all horizons; rather, the separation is less for some [horizons] and greater for others. *48* Similarly, the arctic [circles] do not maintain a distance from the poles that is equal for every latitude; rather, it is less for some and greater for others. However, all the spheres are inscribed for the horizon in Greece. . . .

. . .

### *The Zodiac*

*51* The circle of the 12 signs is an oblique circle. It is itself composed of 3 parallel circles,* two of which are said to define the width of the zodiac circle, while the other is called the circle through the middles of the signs. *52* The latter circle is tangent to two equal parallel circles: the summer tropic, at the 1st degree of the Crab, and the winter tropic, at the 1st degree of the Goat-Horn. It also cuts the equinoctial in two at the 1st degree of the Ram and the 1st degree of the Balance. *53* The width of the zodiac circle is 12 degrees. The zodiac circle is called oblique because it cuts the parallel circles. . . .

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### *The Milky Way*

*68* The Milky Way* also is an oblique circle. This circle, rather great in width, is inclined to the tropic circle. It is composed of a cloud-like mass of small parts and is the only [circle] in the cosmos that is visible. *69* The width of this circle is not well defined; rather, it is wider in certain parts and narrower in others. For this reason, the Milky Way is not inscribed on most spheres.*

This also is one of the great circles. *70* Circles having the same center as the sphere are called great circles on the sphere. There are 7 great circles: the equinoctial, the zodiac with the [circle] through the middles of the signs, the [circles] through the poles, the horizon for each place, the meridian, the Milky Way.[32]

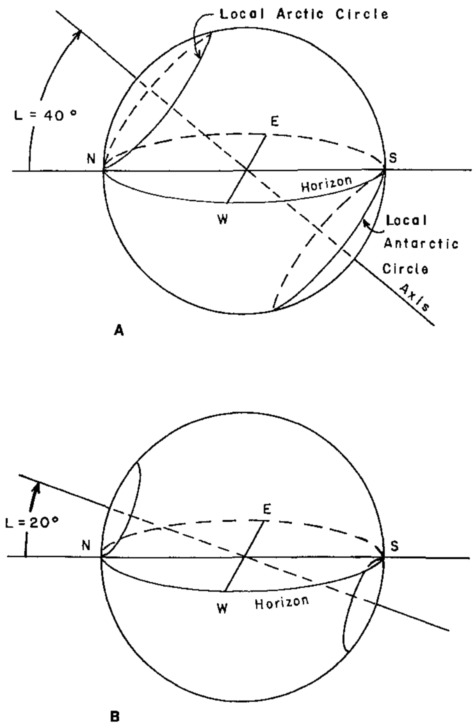


FIGURE 2.13. A (*top*). The local arctic and local antarctic circles in the sense of the Greek astronomers, shown for a latitude of 40° N. B (*bottom*). Local arctic and antarctic circles for a latitude of 20° N.

*Notes to the Extract from Geminus*

*1. Equinoctial.* The equinoctial circle is the celestial equator. It is called equinoctial because the Sun makes the day and night equal when it is on this circle. The Greek term is *isemerinos kuklos* (equal-day circle).

*2. Arctic circle.* For the Greeks, the arctic circle is a circle on the celestial sphere, with its center at the celestial pole, and its size chosen so that the circle grazes the horizon at the north point (see fig. 2.13A). The stars within the arctic circle are circumpolar; that is, they never rise or set. The size of the local arctic circle depends on the latitude of the observer. Figure 2.13A shows the arctic circle for latitude 40° N, and figure 2.13B, the arctic circle for latitude 20° N. The radius of the arctic circle is the angular distance of the celestial pole above the north point of the horizon. But this angular distance (the altitude of the pole) is equal to the latitude of the place of observation. Thus, the radius of the arctic circle for a particular place is equal to the latitude of that place. The modern celestial arctic circle is fixed in size: it is a circle, centered on the pole, with a radius of about 24°. The modern, celestial arctic circle is the path traced out by the pole of the ecliptic during the daily revolution of the heaven. (The arctic circle one sees marked on globes of the Earth can be regarded as a projection of the one in the sky.) Because the size of the arctic circle in the Greek style varies with the location of the observer, we shall call this the *local arctic circle.* The fixed circle of the present age will be called the *modern arctic circle.*

*Oikumene.* The *oikumene* is the *inhabited world.* It is used by Greek writers in two different senses. It may designate the Greeks' portion of the Earth, as opposed to barbarian lands. But the word is also used by geographical writers to mean the whole inhabited world, namely, Asia, Europe, and Africa. Geminus here appears to speak in the more restricted sense.

*Great Bear.* In ancient Greece the foreleg of the constellation of the Great Bear (Ursa Major) did not quite set, but grazed the horizon in the north. The foreleg of the Bear was therefore situated on the local arctic circle, and in the course of the night, it traced out this circle in the sky. Our word *arctic* derives from *arktos,* the Greek word for *bear.*

*5. Tropic.* Our word *tropic* derives from the Greek word *trope,* a *turn, turning.*

*24. Clime.* "Clime," from *klima* (region or zone), but originally a *slope* or *inclination*: the clime is determined by the *inclination* of the axis of the cosmos to the horizon. A clime is a zone of the Earth lying near one parallel of latitude. Often, climes were designated in terms of the length of the longest day. Thus, one might say that Seattle and Basel are in the clime of sixteen hours: at both these cities the length of the day at summer solstice is sixteen hours. The verses that Geminus quotes are from Aratus's *Phenomena,* lines 497–499.

*24. The longest day is 15 equinoctial hours.* In Greece five-eighths of the summer tropic circle is above the horizon. The length of the solstitial day is therefore 24 hours × 5/8 = 15 hours.

*51. Composed of 3 parallel circles.* See figures 2.2 and 2.7.

*68. The Milky Way.* The "Milky Way" is *galaktos* (milky) *kuklos* (circle), from which comes our word *galaxy.* Aristotle, in *Meteorology* I, 8 (345a11–346b15), discusses several theories of the Milky Way: (1) Some of the Pythagoreans held that the Milky Way was a former course of the Sun and that this track had been burned. (2) Anaxagoras and Democritus said that the Milky Way was the light of stars lying in the Earth's shadow. Many of the stars on which the Sun's rays fall become invisible because of the brightness of these rays. But faint stars in the Earth's shadow do not have to overcome the brightness of the Sun's rays and thus they become visible. (3) According to a third opinion, the Milky Way was a reflection of the Sun. Aristotle refutes each of these theories in turn: (1) If the Milky Way were a former, scorched track of the Sun, one would expect the zodiac also to be scorched, but it is not. (2) If the Milky Way were the light of stars lying in the Earth's shadow, the position of the Milky Way should change during the year as the Sun's motion on the ecliptic causes the shadow to move. Besides, the Sun is larger than the Earth and therefore the cone of the shadow does not extend as far as the sphere of the stars. (3) The Milky Way cannot be a reflection of the Sun, for it always cuts through the same constellations, although the Sun's position among the stars is constantly changing. But if one moves an object around in front of a mirror the location of the image is also seen to change. Aristotle's own opinion is that the Milky Way consists of the halos seen around many individual stars. These halos arise in the following way. Above and surrounding the Earth, at the upper limit of the air, is a warm, dry exhalation. This exhalation, as well as a part of the air immediately beneath it, is carried around the Earth by the circular revolution of the heavens. Moved in this manner, it bursts into flame wherever the situation happens to be favorable, namely, in the vicinity of bright stars. Aristotle points out that the stars are brighter and more numerous in the vicinity of the Milky Way than in other parts of the sky. The only objection one might make is that the dry exhalation ought also to be inflamed in the vicinity of the Sun, Moon, and planets, which are brighter than any of the stars. But, according to Aristotle, the Sun, Moon, and planets dissipate the exhalation too rapidly, before it has a chance to accumulate sufficiently to burst into flame. Note that for Aristotle the Milky Way is an atmospheric, and not a celestial, phenomenon: it is produced at the outer boundary of the air.

*68. The Milky Way is not inscribed on most spheres.* However, Ptolemy, in his directions for constructing and marking a celestial globe (*Almagest* VIII, 3), includes the Milky Way among the objects to be represented.

## 2.6 RISINGS OF THE ZODIAC CONSTELLATIONS: TELLING TIME AT NIGHT

In everyday life, the Greeks kept time differently than we do. Rather than dividing the time between one midnight and the next into twenty-four equal parts, they divided the time between sunrise and sunset into twelve seasonal hours, which changed in length through the year as the day itself changed. Similarly, the night was divided into twelve seasonal hours, all equal to one another, but not equal to the day hours (except at the equinox). "Two hours after sunset" meant one-sixth of the way from sunset to sunrise. It did not matter that the time from sunset until the second seasonal hour was nearly twice as long in winter as in summer.

The seasonal hour may seem strange to a modern reader. But nature provides a means of observing the time at night, at least approximately, in terms of seasonal hours. In the course of any night, six signs of the zodiac rise. The proof of this assertion is elementary. At the beginning of night (sunset), the point of the ecliptic that is diametrically opposite the Sun will be on the eastern horizon. At the end of the night (sunrise), the point opposite the Sun will have advanced to the western horizon. The half of the ecliptic following this point is then seen above the horizon and is the very part of the ecliptic that rose in the course of the night.

The risings of six zodiacal signs every night divide the night into six roughly equal parts, of two seasonal hours each. A glance toward the eastern horizon, to see which zodiacal constellation is rising, will suffice to determine the time of night, provided that one knows which constellation the Sun is in.

This information is provided by table 2.1. From March 21 to April 20, the Sun travels from longitude 0° to longitude 30°; that is, it traverses the sign