From: Kepler, The Harmony of the World, trans. Aiton, Duncan, Field (Philadelphia, APS, 1997)

CHAPTER V.

On the Causes of the Influential Configurations, and of their Degrees in Number and Order.

Definition I

The word "configuration" is used for the angle between two rays, each descending from its planet, the angle at which the rays meet here at the Earth (which is deemed to be a point); or, which comes to the same thing, it is used for the arc of the great circle drawn on the zodiac, the arc which is the measure of the said angle; or the arc which the two planets seem to mark out by the interposition of their bodies and, so to speak, cut off for us dwellers on Earth.

It must first be noted about the name, that Ptolemy[43] *in his* Tetrabiblos, *in his* Almagest, *and in his* Harmony *calls "Appearances" what the Arabs translated as Aspects, as if a countenance were the same as a visage or face. Our Teutonic idiom does the same, as it generally names a face "das Angesicht," "aspect," and the countenances of characters who are impersonated, placed in front of the face, "Gesichter," which the Italians call "mascaras" [masks]. However, the word "regards" is also found [in Greek], which not only do we, following the Arabs, translate as "aspects," but also good authors generally call "looks" in Latin, and the signs "onlookers." But in this sense the word is less appropriate for the planets than for the signs or twelfth parts of the zodiac; for as they have length, with their inner side they can be turned towards each other to a greater or less extent. For contiguous signs cannot look at each other, seeing that their faces are not turned towards each other, but are nearly turned in the same direction as each other.*

Second, note that at the same time as two planets make an angle at the Earth they also make other angles at different places in the world; but never the same size as on the arc of a circle passing through the bodies of themselves and the Earth, or rotated about a line which joins their bodies, as if about an axis.[44] *Outside these places their radiations meet at angles which are far different, whether they are also harmonic, or incongruent, as they generally are throughout the rest of the world. Note chiefly that no angle is set up at either one of the planetary bodies. For two rays are required for an angle; but every ray is outside a body, and none is actually in the body. That must be noted*

[43] *Tetrabiblos*, Book I, Chapters 2 and 13, *Almagest*, Book VIII, Chapter 4, *Harmonica*, Book III, Chapter 9.

[44] The angle is the same at any point of the arc described and at any point of the surface formed by the rotation of this arc about the line joining the bodies of the planets.

because the formation of an angle at the Earth is followed by an effect on the Earth also; and from it at the time of the aspects the material of rains and of other occurrences in the sky is exhaled. Hence we can argue of the seat of the cause which sets in motion occurrences in the sky that it is not in either one planet or the other, nor in any empty place in the world, but in the Earth itself.[45]

Third, it was not superfluous to mention that the Earth functions as a point. For the result of that is that although there are innumerable animate beings on the Earth, and an infinity of rays from any planet to them and other points on the Earth, yet the angle between the rays coming from a pair of planets at the same time is sensibly the same at all points of the Earth, whether at the center, on the surface, or in the caverns of the mountains. Although all those aspects are infinite in number, they are taken as a single aspect, seeing that they are all sensibly equal to each other.

Definition II

A configuration is said to be influential when the rays of a pair of planets make an angle such that it is apt to stimulate sublunary Nature, and the inferior faculties of animate beings to be more active, each about its own activities, at the time of the configuration.

Influence is attributed to a configuration, which formally is an entity of the reason, but not immediate influence on the thing itself, as if rains and the like fell from the actual heaven, that is to say from the planets which are in the configuration, which is the foolish persuasion of the common herd, but mediate and objective. For just as objects move the senses, sound the hearing and not the eyes, and color the vision and not the hearing, similarly in this case also, a certain quality of this relation, which is called a configuration, moves not the bodily senses, but a spiritual faculty, capable of reason by instinct without contemplation. Therefore, the configuration brings that about not by its own force but by the force of the soul, which is said to be passive although in actual fact it rather acts itself on itself. After that the sublunary soul or Nature, thus moved or stimulated by the aspect, and reminded of itself, stirs itself up to draw out from the bowels of the Earth the material for every kind of weather. If there were not in the Earth the soul which we call sublunary Nature, the planets of themselves could have no effect on the Earth either on their own account or by means of a convenient aspect. For it is absurd, and like a joke or a flight of poetic fancy, to suppose that from the coming together harmonically of a pair of rays, as if from the coupling of a man and a woman, a vapor is conceived as the material for wind or rain, as if indeed, as the seed is of the substance of the parents, so the moisture and other things which are exhaled from the Earth are of the substance either of harmony, which is a relation, or of an angle, which is a qualitative quantity, or even of light itself, which is a quality and

[45] Thus the influence of the aspects is in no sense a direct physical effect of the rays from the planets, a point that Kepler reiterates in his definition II.

by no means a substance. In fact in the same way as we say that nothing can come from nothing, so also, naturally, nothing material can be drawn from something immaterial. See further Chapter VII.

Axiom I[46]

The arc of the zodiac circle, which is cut off by the side of a figure or of a star[47] which is congruent and knowable, measures the angle of an influential configuration.

Axiom II

The angle of a figure or star which is knowable and congruent is the gauge of the angle of an influential configuration.

The whole business rests on the two axioms; and the reason why I have made them two is that there are two probable means by which souls and sublunary Natures can come to knowledge of the configurations which exist at a given time.

For they either perceive the figure of which a side cuts off an arc of the zodiac circle as the measure of the configuration or of the angle of the rays, or the figure of which the actual angle of the configuration is an element. What the distinction is between the figures, and what on the other hand the affinity is, is evident to the eye from these diagrams, which follow each other in order from this point. For the pairs of diagrams are "matched," first, a diameter with itself, whether it bisects its circle, being drawn through its center, or touches it, when the two rays make as their angle two right angles, or rather no angle, being arranged in a single straight line.[48] *The same must be supposed of the conjunction of planets, either superior, ☌, or inferior, ⚲, which is improperly called a configuration.*[49] *For if two planets were at the same point on the zodiac, then there is no angle at the center, on the circumference an infinite number,*

[46] The purpose of these axioms and the propositions which follow is to give a rational basis for the number of aspects and to order them according to their degree of influence. The two axioms resemble that which relates knowable polygons and the musical consonances in Book III, Chapter 1. These new axioms, however, give two forms of relationship between the sensible harmonies and their geometrical archetypes, through the arc cut off by the side and through the angle between two adjacent sides. Both axioms, as Kepler points out, lead to the same set of aspects. The advantage of retaining both is that they provide a theoretical basis for the fine ordering of the aspects in terms of their degree of influence. Each aspect is related to two polygons, the central one and the circumferential one. In the development of his theory, Kepler establishes the relative importance of the central and circumferential polygons for their corresponding aspect and decides which properties of the polygons should be considered as determining the properties of the aspects.

[47] That is, a polygon or star-polygon.

[48] That is, in the case of opposition (Fig. 1), the reciprocal figure (not shown in the diagram) is a circle touching the diameter at the center.

[49] Ptolemy did not count conjunction as one of the aspects, though he always treated it as if it were an aspect.

and the sides of the figure are points: that is to say, the circle is like a figure with an infinite number of angles. This configuration had no need of visual representation.

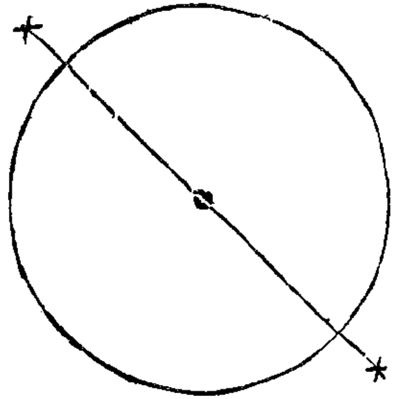


Figure I.

Second, the square is also "matched" with itself, because the angle which two of its sides form at the circumference is equal to the angle at the center subtended by one side. After that the triangle is "matched" with the hexagon, the pentagon with the ten-angled star, the octagon with the eight-angled star, the decagon with the five-angled star, and the dodecagon with the twelve-angled star, in such a way that if all the angles of one member of a team are drawn at the circumference, its side subtends one angle, located at the center, of the other member.

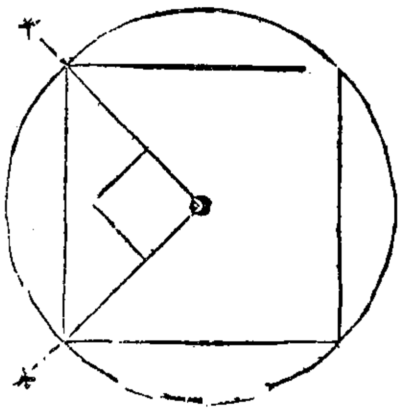


Figure II.

Now the centers of all the circles suggest the Earth, as if it were placed in the middle, and the circles themselves represent either the zodiac, as pictured from the Earth, or whatever other circle, subordinate to the zodiac, is pictured in order to measure the angles. Such circles are potentially the actual souls which are moved by the aspects, that is circles abstracted as it were from actual quantity and constrained within the restrictions of a point which has qualities and is capable of having directions.

However, I have placed two radiant stars outside this circle, one higher than the other, to convey by a visual representation that it makes no difference to the configuration at the Earth whether the planet is high or low in the heaven, and one can be many times higher than another while the configuration at the Earth remains the same.[50]

Furthermore, it came about not by accident but on purpose that in the first axiom the word "congruent" comes first, in the second the word "knowable." For both figures share the responsibility for some aspect's being influential, the one which is drawn at the circumference as well as the one of which one angle is formed by the rays at the center, both of them on account of knowability as well as on account of congruence; yet they are not equally responsible. All that de-

[50] In other words, the aspect is defined not by the angle between the actual rays coming from the planets but by the angle between their lines of longitude.

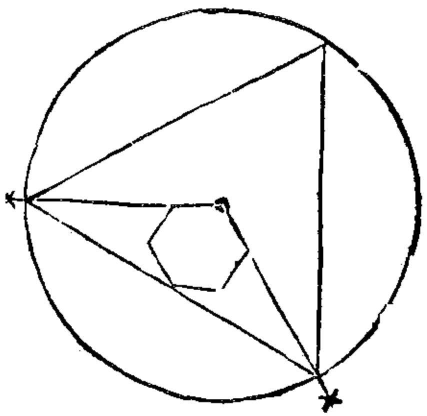
mands a somewhat more extensive explanation; and there is a single reason why it is so prickly—so that the number of aspects can be reduced by philosophical arguments, or at least can be differentiated into definite degrees. For if I had been willing to include four additional aspects along with the usual eight, this discussion could have done without several, so to speak, propositions which follow, seeing that they are taken up only with comparison.[51]

Proposition I

The affinity between the rays and a circle and its arcs is greater than was that between the consonances.

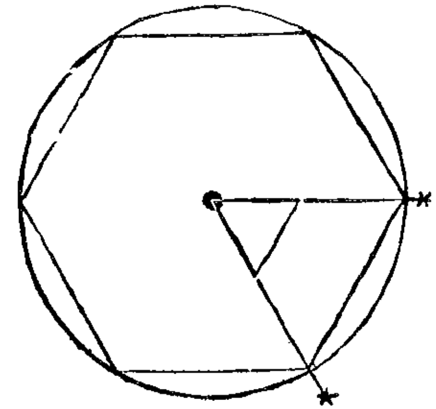
The bases of what was previously stated in Book III were already justifiably taken over into Book IV as an axiom. However, it is proved as follows:

Figure III.



1. Consonances are between sounds; sounds consist in motions; their heights or depths in pitch, by which the consonances are expressed, arise from the speed and slowness of the motions, by what has been demonstrated in Book III. But the quick and slow sounds are elicited from the striking of strings which are stretched, not only if the stretching is in a circle, but also, and much more, if it is in a straight line. Therefore, the consonances do not relate immediately to the circle and its arcs, on account of their circular shape, but on account of the length of their parts, that is their mutual proportion; and they have what they have from the circle even when the circle has been destroyed and stretched out into a straight line. Aspects on the contrary are by Definition I angles, which the circle measures along with its arcs only if it remains what it is said to be, that is insofar as it both has its circular shape and retains it completely.

Figure IV.



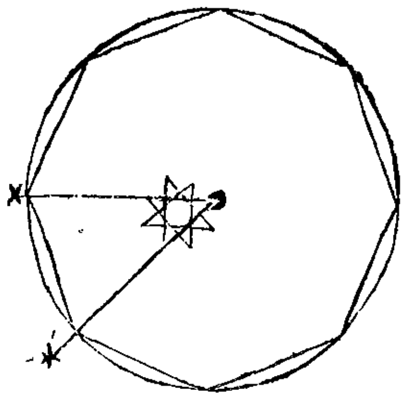
2. The consonances were not all propagated with an equally close relationship to the circle and its parts; for some of them related their origin to parts of the circle, insofar as they had some property not as parts of the circle but as straight lines, that is the same division as the whole circle, as was shown in Book III. It is the opposite in the case of the aspects; for the measure of what corresponds with the circle cannot in any way be related to straightness.[52]

Proposition II

The affinity between the rays and the regular figures is greater than was that of the consonances.

It is proved first by the figure at the circumference. For where the circle is complete, in that case the regular figure is also complete. But the circle is more complete in measuring the angles of rays, by I. Then the figure also can be considered more as complete in relation to the rays. It is the opposite in the case of consonances: as the circle and its parts could be stretched out straight, while preserving the consonances, so also all the sides of a figure could be stretched out into one and the same straight line, and could make a consonance with one straight side of the figure. On that

Figure V.

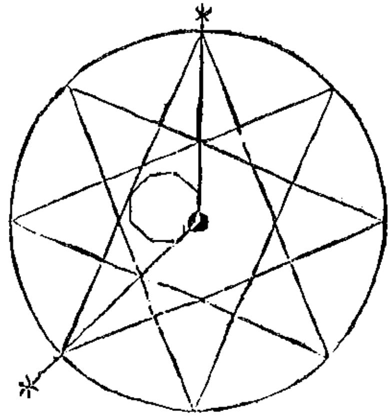


[51] Ptolemy (*Harmonica*, Book III, Chapter 9) recognized only four aspects; namely opposition (180°), trine (120°), quadrature (90°), and sextile (60°), though, as noted above, he always treated conjunction (0°) as if it were an aspect. On the basis of weather observations, Kepler (KGW 16, p. 114) in 1608 defended the three additional aspects which he had accepted in 1599 (KGW 13, p. 349), namely quintile (72°), biquintile (144°), and trioctile (135°). This brought the total number of aspects (including conjunction) to what he called the usual eight. See Field (1984a), pp. 202–204.

Ptolemy identified the astrological harmonies, or aspects, with the musical harmonies, for both, he claimed, were derived from the division of the circle into two, three, and four parts. Moreover, if the circle was divided into twelve parts, twelve being the smallest number to have half, third, and fourth parts, the resulting sections or zodiacal signs corresponded to the tones (approximately twelve in number) of the double-octave scale (*Harmonica*, Book III, Chapter 8). In the notes on his translation of Ptolemy's *Harmonica* (KOF, vol. 5, pp. 372–375), Kepler shows that Ptolemy is inconsistent, even according to his own principles, in accepting the sextile as an aspect but rejecting the semi-sextile (or duodecile). Moreover, he demonstrates to his own satisfaction the insufficiency of Ptolemy's foundations and claims that a true theory of the aspects can only be built on the geometrical properties of the division of the circle. Kepler develops his own theory of the aspects in the following propositions, showing that they have a different origin from that of the musical consonances. It follows that, although they are related, the two types of harmonies—musical consonances and aspects—are not identical.

[52] The mixture of physical and mathematical reasoning employed here is characteristic of most of the following propositions, though the relative importance of the physical and mathematical components varies from one proposition to another.

Figure VI.



basis, indeed, just as the circle loses its configuration, so also does the rectilinear figure, so that it is no longer a figure.

It is proved second by the figure at the center. Angles are the elements of figures. In this case two rays make the angle at the center; and if that is repeated a certain number of times it completes the figure, as is apparent from these diagrams. However, that was not the case with the origin of the consonances: for there was no relationship with the angle at the center in that case. Therefore, the figures are more closely akin to the aspects than to the consonances.

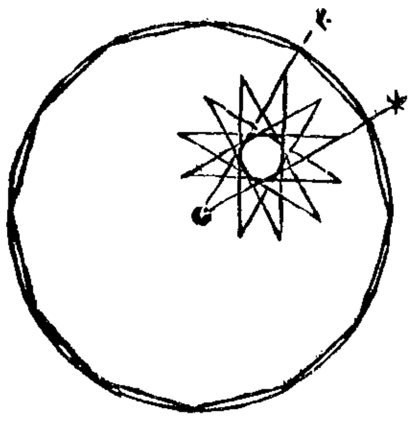
Proposition III

The congruence of figures is capable of more in establishing influential configurations than in the case of consonances.

Many arguments for this point are forthcoming. 1. Congruence is a property of a figure insofar as it is a whole figure and has an appearance. But a figure, insofar as the whole of it has this appearance, first, in itself on its own account has more affinity with configurations than with consonances, by Proposition II. Second, it divides the circle as a whole harmonically; but the circle also has more affinity with configurations than with consonances, by Proposition I. Hence on both showings, on that of the figure independently and that of the figure and the circle in common, the force of the congruence of figures is also greater in configurations than in consonances.

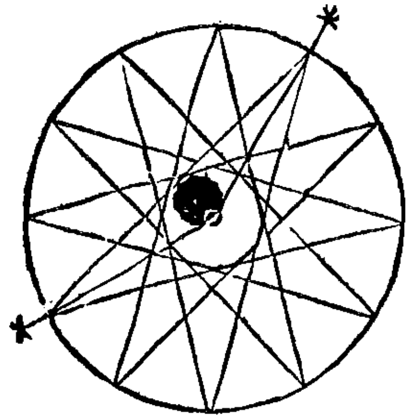
2. From the number of figures. For we have taken it, on account of the axioms previously stated, that figures are influential on account of their properties. Then where the number of the things influenced corresponds more, there is a greater affinity between cause and effect, at least probably. But as the congruent figures are few, so also the aspects are few, as experience testifies. For if they were not few, there would be a great confusion of them, and a great multitude, so that individual aspects could not be observed separately on their own days. But they can be observed: therefore, they are not infinite in number. On the contrary, the consonances can be infinite, by augmentation of the intervals by

Figure VII.



diapasons, just as the knowable figures are infinite.

Figure VIII.



3. From the essence of the terms, in which the proportions in either direction have their existence. Motions, of which sounds are a disposition, are considered as Becoming, insofar as they take time; radiations more as instantaneous Being. For just as a body exists at this moment, so also a radiation exists at this moment; whereas of motion what has passed no longer is, what follows is not yet, in a moment it is nothing. Congruence, however, seems to be among things which are, rather than among those which become. For the sides or wall of a house are congruent so that the house may exist, not so that now and perpetually it may be built.

4. From the affinity of congruence, as cause, with configurations. For the latter are angles; but congruence also exists in figures on account of their angles.

So far consonances and configurations have been opposite to each other. In what follows there will be another opposition among configurations alone, between congruence and knowability.

Proposition IV

The congruence of figures is capable of making configurations influential more than knowability.[53]

It is proved from the circumstances of the sublunary Soul and also of the faculties of the human soul which perceive the aspects. These, then, are all inferior to the contemplative faculty and the understanding; and they have closer affinity with the sensitive faculty and the one which controls the operations of the senses. Indeed, the instinct of the former, as we have stated in Chapter III, is as much duller than the human instinct as the body of the Earth is cruder than the body of Man. But congruence is later in order than knowability, and is, as it were, presented outwards to some operation which has the idea of the operations of the senses. It is therefore right for it to be believed that these animal faculties are moved and affected by congruence rather than by the knowability of the figures.

Congruence and knowability were therefore opposites in the same kind of configuration. Now two figures will be opposed to each other, first with reference to congruence alone, after that with reference to configurations.

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[53] The reasoning in this proposition is almost entirely physical.

Proposition V

Congruence is a property of the figure at the circumference rather than at the center.[54]

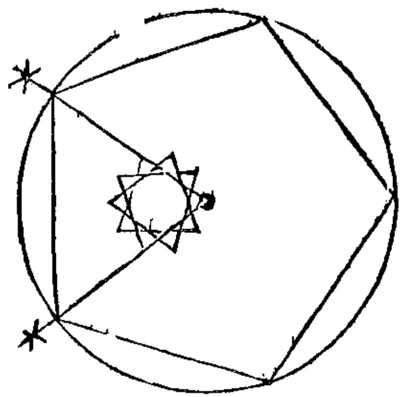
For it has the more important position in the figure which can be whole, on account of the position from which it takes its name, because congruence belongs to whole figures, as is apparent from Book II. But of the central figure not more than one angle can stand at the center, by Definition I. However, at the circumference the whole of the circumferential figure can exist. The proposition follows.

Proposition VI

Of the two figures which each aspect claims for itself, that at the circumference is more important than that at the center.

For congruence is more important in this business than knowability, by IV. But congruence is more important in the figure at the circumference, by V. Hence, that which has superior power also makes the very thing in which it is have superior power, that is to say the figure at the circumference.

Figure IX.



The same is also proved in the following way from the inmost properties of the soul, touched on in Chapter III. For as it is the soul which adapts its own formal essence to the harmonies of the configurations, certainly by the same distinction by which the soul is either a circle,[55] or a point which is the center of a circle, so also will the figures which belong to its household, those at the circumference and at the center, be distinguished.

In fact, every soul bears some idea of a circle, in fact of a circle abstracted

[54] Here the reasoning is entirely mathematical. Kepler asserts, quite reasonably, that the capacity to form congruences is a property of the figure as a whole and the circumferential polygon is employed as a whole (the circle passing through all its vertices), whereas the central polygon has only one of its angles at the center. This difference enables Kepler to justify the use of the hierarchy of polygons defined in terms of congruence as a means of determining the relative status of the aspects, once he has shown, in proposition VI, that the circumferential polygon is more important in the determination of the aspects, and in proposition VII, that for the circumferential polygon, congruence is more important than knowability.

[55] The symbol for the soul is a circle, of which the circumference represents its external and the center its internal manifestation. In its passive (perceiving) role, as when it perceives the aspects, Kepler thinks of the soul as a point but in its active (operative) role, when it responds to the stimulation of the aspects, he thinks of it as a circle. As his prior concern is for the influence of the aspect rather than for the way in which it is perceived, Kepler gives more attention to the circumferential polygon than to the central polygon.

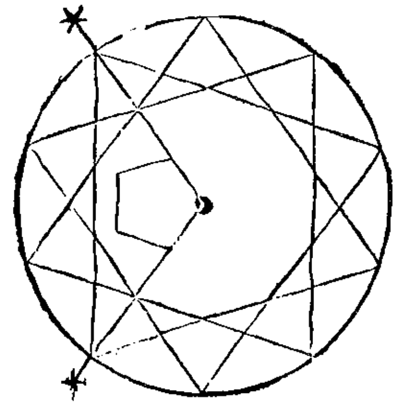


Figure X.

not only from matter but also in a sense from magnitude, as was stated in Chapter III, and so the circle and its center in this case almost coincide, and the soul is either a potential circle or a point in which directions are distinguished, and thus it can be said in a sense to have qualities. Yet it seems that the division must be observed between those faculties of the soul which should be considered rather as a circle, and those which should be considered rather as a point. For just as a circle cannot be imagined without a center, and on the other hand every point has round it space for drawing a circle. so also in the case of the soul there is no operation without an image-forming impression, and on the other hand every internal reception or meditation is on account of an external motion, and very interior faculty of the mind is on account of those which are rather exterior. What is the chief and supreme faculty of the soul, called the mind, if not a center? What is the reasoning faculty if not a circle? For just as the center is inside, and the circle more outside, so the mind keeps itself to itself, and reasoning weaves a sort of exterior web; and just as the center is the basis, source, and origin of a circle, so the mind is of reasoning. Again, every such faculty of the soul, both the understanding, the contemplative, and indeed the sensitive faculty, are a sort of center. But the faculties of the soul which produce motion are a circle, because again, as a circle on the outside is placed about its center, so operation is towards the outside, and cognition and meditation are performed within; and as a circle is to a point, so in a way external action is to internal contemplation, animate motion to sensation. For a point, because it is opposite to the circumference in all directions, is born suitable for representing passive experience; and for what is the sensitive soul,

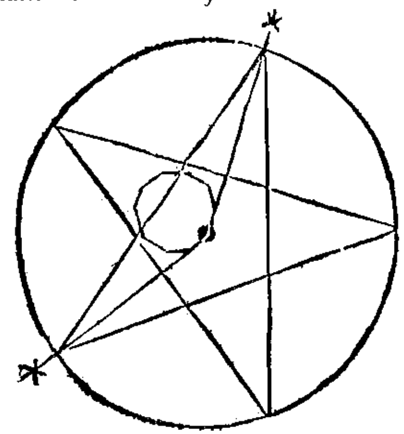


Figure XI.

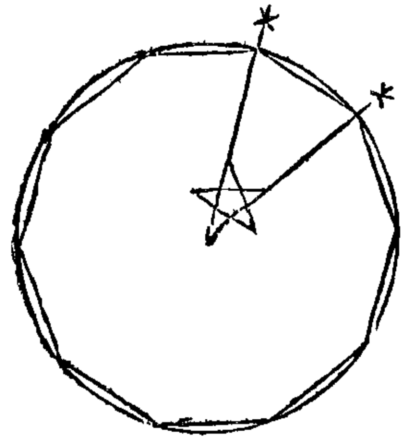


Figure XII.

even as in this case when it is that which perceives radiations, except for sensing and perceiving what it experiences passively? That is, because it is moved by its objects.

Also by comparing each point of comparison: as the center is the same in either case, so also the form of cognition is in a way the same, the chief mental one, and the sensitive, or, analogous to it, that which perceives radiations: neither uses contemplation in itself, insofar as it is what it is, but it recognizes without contemplation. Hence, as the former, I mean sublunary Nature or even the sensitive faculty, is a sort of faint image of the latter, the chief faculty of the human mind, so the former, contemplation by the reason, is an image of these actions and operations of the soul, a circle in each case.

Therefore, insofar as souls perceive celestial radiations, and so are to speak moved within themselves by them, let them be for us points; but on the other hand insofar as they are movers, that is, insofar as they transfer perceived harmonies between the radiations into their own operations, and are stimulated by them to action, they should be considered as circle.

It therefore follows that to the extent to which it is recognizing the harmonies of the rays, it is concerned chiefly with the central figure; whereas to the extent to which it is operating, arousing occurrences in the sky (and similar ones in Man), it fits itself to the one at the circumference. And in fact in the case of an aspect, we give prior concern for its influence than for the means by which it is perceived by the operative soul, and therefore, also prior attention to the figure at the circumference rather than to the figure at the center.

In this case, figure was compared with figure in one and the same aspect. In the following couple, supposing the figure is the same, its congruence will be opposed to its knowability.

Proposition VII

In the figure at the circumference, congruence takes precedence over the knowability of the side. In the figure at the center on the other hand, the knowability of the side takes precedence over the congruence of the figure.

This proposition pertains to the completion of the matter which was left half finished in Proposition III. For although congruence is more important in IV than in III, constructibility was also more important in Book III. Yet this should not be completely separated from the constitution of the aspects, since no congruence is without a knowledge-producing construction; and on it depends that of both the side and particularly of the area bounded by the sides of the figure. For it is from the angles, in which suitability for congruence resides, that the construction is derived.

It seems, therefore, that as far as this proposition is concerned its contrary is true, both in the one case and in the other. For as far as the figure at the center is concerned, one angle of it has in actual fact been expressed through the rays; but of the figure at the circumference no angle is expressed, but only its side, up to a point. However, congruence belongs to angles. Therefore, it seems that we must take most notice of that in the central figure. For if sublunary Nature perceives the quantity of an angle, which two rays form at the Earth, it seems that it can also perceive the fitness of that angle, along with the others, for congruence.

The fact that on the other hand we should take more notice in the figure at the circumference of the knowability of its side than of its congruence seems evident from the following. For the knowledge by which a figure is known, as was demonstrated in Book I, is based on the equality either of a side to an expressible part of the diameter, or of the square of a side to an expressible part of the square of the diameter, or of the area of the figure to the same, or on another linkage of either the side, or of its square and area, to the diameter or its square, or determination by it. Suppose, therefore, that sublunary Nature has sensation of the zodiac circle, a circle which is of course sensible, and stands outside it, which it examines against the idea of an abstract intellectual circle, which it has within itself and born or created along with itself. In that case it certainly follows that in the natural order it first senses the size of the arc of the zodiac marked off between two planets, the size of the chord which it subtends, its kind, whether it is expressible in length or only in square, whether it makes an expressible sum of squares in combination with any other, and an expressible rectangle, and on what property the expressibility of its area is founded. These things, I say, seem to become known to sublunary Nature first in the natural order, because a side is prior to a figure which is described by a repetition of the side; and it is only afterwards when the whole figure has been described on the zodiac circle, that the angles become evident, and their quantity, and whether they are among the congruent, and whether the figure converges with all its angles to the same kind of congruence, or whether the congruence is continuous. In short, congruence is a feature of angles, knowability of sides. Therefore, when the angle of a figure becomes known before its side, in that case the parts of congruence seem to be earlier and more important than those of knowability; when it is known after, they are later. But it is the angle of the figure which reaches the center which becomes known first, whereas the side of the figure described at the surface (I mean between only two planets) becomes known first. Therefore, it seems to be right if in the figure at the center consideration of congruence is held to be more important, but in that at the surface that of knowability.

We must therefore rebut these points which have been adduced to support the contrary of our proposition, and in the same operation we must reinforce by true arguments the order of the properties of congruence and knowability.

First, then, although it is true in respect of the central figure that one angle is formed by the rays of two planets, yet it does not follow from that that the mind when it perceives the quantity of the angle perceives first in the natural order the congruence of the figure to which the angle is going to belong. The reason is evident, that congruence, insofar as it belongs to one angle, and several equal to it, placed in a single plane, is too general. For there are infinite forms of angles which are congruent in that way, and they are always greater in number in proportion as the individual angles are smaller. This, then, is not the congruence which we discuss in Book II which is a property not of angles indi-

vidually but of complete figures on account of their angles, and not of single figures but of several in combination.

Thus, the objection is not only rebutted but turned against itself. For the assumption it had made about the figure at the circumference, we can with equal justice make about that at the center, that congruence, as was stated in our proposition here, is in it later than knowability, so that the part it plays ought to be more important than that of the former, by our adversary's own admission. For a figure must come into being before the whole of it can be congruent. However, in this instance unless the side of the figure is knowable, the figure cannot come into being. For although it is true that if in a figure one angle is given, which the rays of two planets form at the center, the number of all of them is given, and through them the fitness of the whole figure for congruence, and the nature of the side does not enter into this demonstration, yet that one angle of the figure is not given, that is, it is not recognized as an angle of a congruent figure, except through a knowable side. Therefore, the soul knows the side before (in natural order) it recognizes that it is given a congruent angle.

Figure IV.
Figure III.

But if we compare both figures in this case, the side or area of the figure at the center is given less, in fact, by the actual radiation, than the side of the figure at the circumference. For the latter is always bounded by the actual rays, the latter not always, but only in certain figures, as in the triangle, because the individual sides of the angle at the circumference subtend an arc equal to the intercept. Therefore, the figure at the center is more remote from the act of knowing than that at the circumference. Therefore, the recognition of congruence is also more remote. However, the objection rested on the contrary, as if the congruence of that at the center was more accessible to recognition than the congruence of that at the circumference.

Further, if the quantity of the angle is perceived, by what will it be perceived if not by its own measure, that is, by the arc of that circle which is drawn from the point of the angle in question, that is the Earth? But not at first by an arc of that circle which is drawn round the central figure, and which passes through the Earth. Hence in the perception of the quantity of an angle in the central figure, it must as a soul show that power by which it is a circle, not that by which it is a tiny point, to which the angle extends. But by the same character of its circular essence it also perceives the side of the figure at the circumference, and its arc, and that at first. Only after that, with the doubling of this arc, does the arc of the smaller circle also emerge, which is drawn round the central figure, and passes through the center of the previous one. This arc assists the inscription of the figure at the center in the circle. For the order which occurs in reasoning is also the same in instinct. Again, then, a way is demonstrated to the perception of the central figure, a longer way, and so also to its congruence. Hence the objection reverses itself, as it based its superior force on priority in perception.

To the reasoning for the other branch of the argument the following reply has to be made. In fact even in the figure at the circumference the knowability of the side is in truth prior to the congruence of the whole figure, on account of the arguments stated, which are also valid in this case. But it does not follow that of two things, one of which is the cause of the other, the one which is the cause also in addition moves a third one more strongly. For in relation to the grasp of the soul, which has to be moved, often for that purpose the cause has less force than the effect. So in this case the sublunary Soul, insofar as it is indeed perceptive, is more moved by the knowability of the figure at the center; whereas insofar as it is operative, it is more moved by the congruence of the figure at the circumference.

However, my own arguments to prove the proposition are the following. We first took it in Proposition VI that the center is a sort of idea of the speculative mind, or understanding, the circumference of the practical or operative faculty, because as the center is the basis and origin of a circle, so meditation is of action. Then the figure which offers its angle to the center, that is to the Earth, where the soul, the perceiver of the figure, has its seat, offers itself, so to speak, for knowing and judging, since the center represents the judgment seat of knowledge. Therefore, in the figure at the center more notice should be taken of knowledge, notwithstanding the fact that it is obtained by means of the circle as if by an instrument, as was stated a little earlier.

On the other hand, the figure which arranges the angles at the circumference applies itself more to imitation and to expression in the operation of the soul, as if working towards the idea of the operations. But it is congruence rather than knowability which has the idea of sensible operations and constructions, because it is a consequence of whole figures, while the side by which the figure is known is merely an element of it. Hence in the figure at the circumference we must have regard to congruence rather than to knowability.

Another argument for this second part depends on the same consideration of the soul. It prevails on account of what the others are. But configurations are perceived on account of the operation of sublunary Nature, and indeed of the inferior faculties of the human soul. That is to say, they are perceived for the purpose of expressing them operationally. The status of the motor faculty is therefore greater in this business. But the knowability of the figure at the circumference assists perception, and its congruence assists operation, as hitherto. Therefore, the congruence of the figure at the circumference also prevails over the knowability of the same.

Proposition VIII[56]

The arc of a circle which is established by an incongruent figure imparts no influence to the rays of two planets which delimit the arc.

For if congruence is the most important cause of influence, by Propositions III, IV, and VII, therefore if that is lacking, knowability will not be sufficient as a more humble substitute cause. For although the latter is more powerful than congruence in the figure at the center, by the other part of Proposition VII, yet on the other hand the figure at the circumference is more important than the

[56] This proposition is a partial converse of axiom I. It is not clear why Kepler did not make the proposition an axiom, though its usefulness in explaining the small number of aspects (notwithstanding the infinite number of knowable polygons) probably led him to regard it as too important to be asserted without proof.

one at the center, by VI. And in that congruence is more powerful, by the first part of Proposition VII. Therefore, the congruence of the figure at the circumference is still more powerful than the knowability of the figure at the center.

See, here is the reason why, whereas the knowable figures are infinite, and indeed of various degrees, the aspects, however, are few.

Axiom III

The arcs of a circle, of which the figures are powerful in more, and more important, degrees of congruence and knowability, also take the more influential configurations.

If the first two axioms are generally agreed,[57] *so will this be, because on account of the fact that every individual one is like that, if the former is intensified, the latter will also be more like that. Understand this, however, in the following way: that in the figure at the circumference the comparison of the degrees of congruence takes precedence, in that at the center comparison of the degrees of knowability, and in fact the part of the figure at the circumference is more important.*

Proposition IX

The influential configurations are those which intercept the following arcs of the zodiac circle:

180°: opposition ☍ from the diameter of the circle, as in Figure I.

90°: the quartile □ from the tetragon, as in Figure II.

120°: the trine △, and 60°: the sextile ⚹, from the triangle and the hexagon, as in Figures III and IV.

45°: the octile or semiquartile, and 135°: the trioctile or quartile-and-a-half ✳, from the octagon and its star, as in Figures V and VI.

30°: the semisextile ✺, and 150°: the quincunx (five-twelfths), from the dodecagon and its star, as in Figures VII and VIII.

72°: the quintile ⊥, and 108°: the tridecile or quintile-and-a-half, from the pentagon and the decagonal star, as in Figures IX and X.

144°: the biquintile ⛤, and 36°: the half-quintile or decile, from the pentagonal star and the decagon, as in Figures XI and XII.

That these figures are knowable and constructible has been shown in Book I: that they are also congruent, in Book II. However, that the configurations of arcs expressed by such are influential is in the Axioms I and II previously stated.[58]

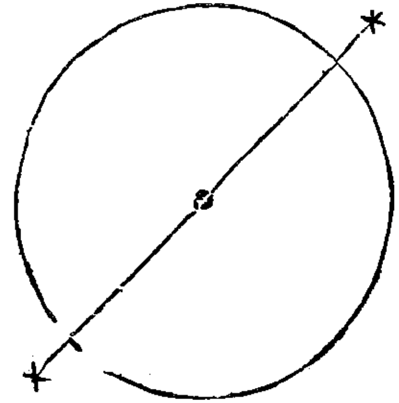
[57] It seems that the word "axiom" does not mean to Kepler quite the same as it does to a modern mathematician. In this chapter he seems to regard the axioms as working hypotheses, which he believes to be true, and wishes to justify if possible. For example, the series of propositions that follows the statement of the first two axioms appears to have been at least partly designed to justify accepting the axioms as true.

[58] Having stated the aspects which are established by the axioms, Kepler, in the propositions that follow, orders them, on the basis of the nobleness of the corresponding polygons demonstrated in Books I and II, according to their degree of influence.

Proposition X

Of the influentiality of the aspects the first and strongest degree is that of conjunction ☌ and opposition ☍.

For in conjunction the two rays are congruent on the same line, and descend from the same side: in opposition ☍ they descend from different sides indeed, but are nonetheless parts of one continuous line. This in fact is the most perfect congruence and a kind of basic principle of all congruence. Thus as conjunction is represented by a point marked on the circumference of the circle, but opposition by the diameter, these are certainly basic principles. The former is also the measure of all knowledge in this class, as all knowledge of a straight line in a circle is contained in the constructible demarcation of it by means of the length or square of the diameter, as was made clear in Book I. Therefore, by Axiom III the basic principle of influentiality is also in these aspects.



The conventional symbols.

Proposition XI

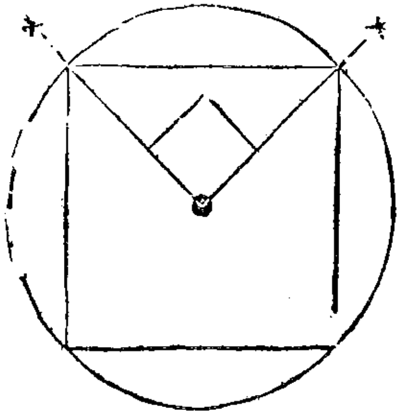
The second degree of influentiality of aspects is that of the quartile □.

For in the quartile many privileges coincide, the first of which is that the figure at its center is similar to that at its circumference. Hence whatever degrees it holds in congruence and knowability are understood to be in a sense double relatively to the other aspects. For just as the quartile is the first, after opposition, to be unfolded from the narrowness of its line to some breadth or surface extension of its tetragonal area,[59] *so the remaining aspects descend from the identity of the figures of the quartile aspect to some difference in their figures. Therefore, as elsewhere in natural philosophy, excellence which is unified is stronger, in this ideal and objective portrayal the degree of strength will be greater where figures in distinct positions, that is one at the center and one at the circumference, are found to be the same in kind.*

Second, as far as congruence is concerned, in the square it is most perfect and of all kinds, for this figure is itself congruent with itself in the solid to form a cube, which is the measure of all solidity; and is congruent in the simplest way, taking in only three angles; and is itself congruent with itself in the plane, with four angles. Again, it is congruent in the solid with the triangle, pentagon,

[59] Kepler evidently means that the side of the polygon, corresponding to the quartile, can trace out the surface or area of this polygon, clearly a property belonging to this aspect alone.

□



hexagon, octagon, and decagon in different ways, to form solid figures; and is congruent with all of those, and in addition with the dodecagon, and the icosagon to a certain extent, to lay out a flat surface. In this property it is surpassed by no other.

Third, the area of the square is expressible, which is the basic principle of a certain singular and outstanding congruence in the plane, so that a definite number of areas of this figure take up a definite number of squares on the diameter, and thus the figures are not only in themselves congruent with each other in their angles and sides, but in a way, that is in definite lines of theirs, also with the sides of the square on the diameter. This property the quartile aspect shares partly with the half-sextile alone. See Book II.

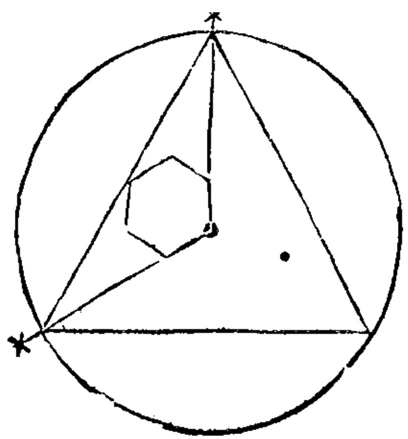
Fourth, the degree of knowledge of the side is also not ignoble, as it is expressible in the square. In this degree it is pre-eminent over all the other figures, except the hexagon. However, it does not on that account give way to it, since knowability is not to be compared with congruence, as has been explained above, and indeed the accumulation of privileges is valid for increasing the influence, by Axiom III of this Chapter.

Proposition XII

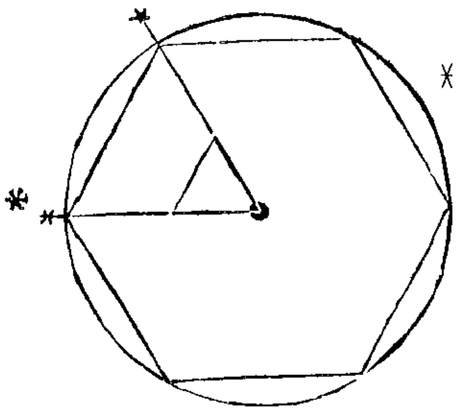
The third degree of influentiality is that of the trine △, sextile ⚹, and semi-sextile ⚺.

That I place the trine, sextile, and half-sixth in the same degree is not due to identity of properties but to equal power. First, their principal figures transmit shared operations to congruent planes; for they meet each other, and others, such as the square, in various ways. In fact, the triangle and hexagon are pre-eminent in this case, because the individual kinds are congruent also with themselves. The hexagon is pre-eminent over the triangle, because it holds the most perfect congruence in the plane, that is in its three angles alone. Both are pre-eminent over the dodecagon, because they are congruent with other figures even in the solid, which the dodecagon cannot be. Yet on the other hand

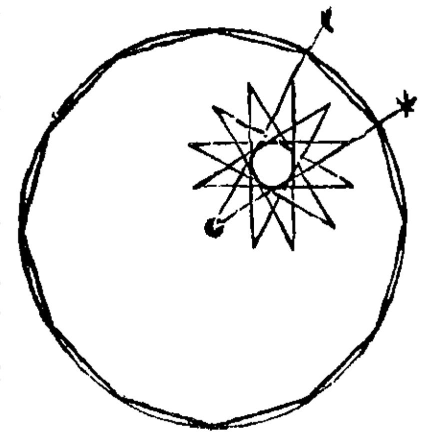
△



the dodecagon is pre-eminent over the others in the expressibility of its area, whereas their areas are medial[60] *and so more ignoble. This difference between the areas, as I stated just now, redounds to the perfection of its congruence. Similarly the triangle also is again pre-eminent over the hexagon, in that the triangular kind is itself congruent with itself in the solid in various ways, and begets three regular solids: the hexagon is congruent only with other figures. Thus on balancing the weights of the different properties against each other, congruence, which is the first and chief element in influence, in the case of these three very nearly produces equilibrium. In knowability the hexagon holds first place, as its side is expressible; second place is held by the triangle, for it takes the same degree as the tetragon, having its side expressible in the square, but in a more humble proportion. The dodecagon is last in this instance, having an inexpressible side. However, knowability is not the chief indication of influence, and is not considered in the most important figure, that is the one at the circumference, but only in the one at the center which is less important. If it has any effect, it renders the trine a little more influential than the sextile, because the trine is formed by an angle of the hexagon at the center, and the half-sixth a little less influential than either, as it is measured out by an angle of the dodecagonal star at the center. However, knowledge of the half-sixth is more noble than the rest which follow, because the side of the figure at its center is of the most noble kind among the inexpressible, that is the binomial,*[61] *and in their twofold subdivision it always holds the superior place, to the extent that along with its partner the side of the figure at the circumference it forms an expressible triangle. That is the mark of almost absolute perfection, and also in fact makes this figure in knowability a contender with the triangle and the hexagon, because it weighs in the balance against its inexpressibility, very heavily.*



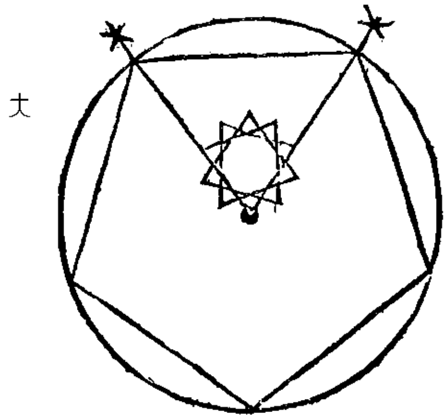
⚺

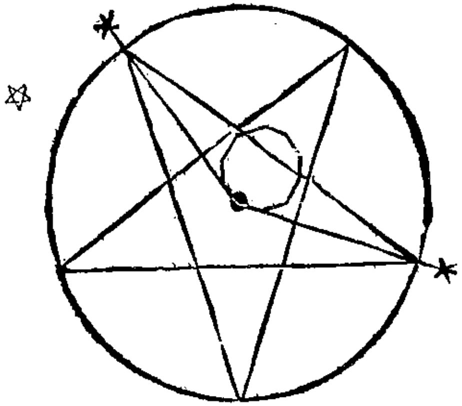


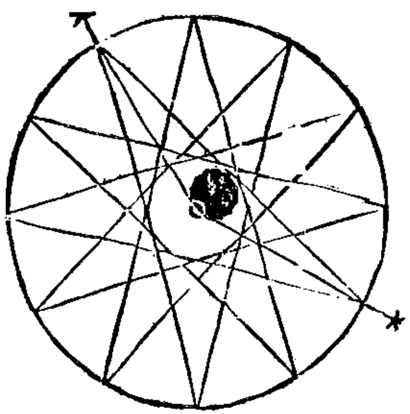
[60] See Book I, definition 15.
[61] See Book I, definition 22.

Proposition XIII

The fourth degree of the configurations in influentiality is that of the quintile, biquintile, and quincunx.







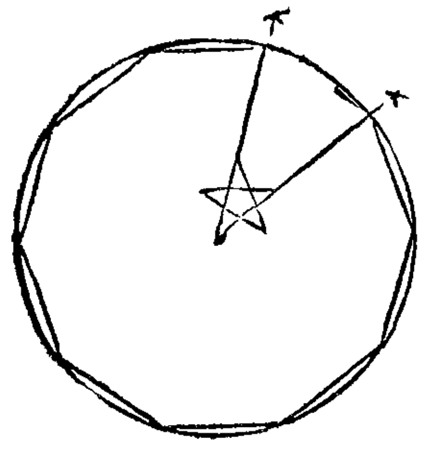
For these have in common congruence of the whole of the primary figures in the plane, though not of the individual kinds each with itself, but of the first two figures mutually with each other, and of the last with the others which are akin to it. The first two aspects are pre-eminent in virtue of the fact that their figures, the pentagon and its star, are congruent in the solid as well, and make two regular solid figures. By this nobility they almost bring their aspects to the level of the trine and quadrant. The dodecagonal star is not congruent in the solid. Yet on the otherhand the dodecagonal star is also pre-eminent by its plane congruence, which in it is capable of being continued to infinity, whereas the former figures cannot be continued far without an admixture of irregularity. See all this in Book II.

As far as the knowability of the sides in the figures at the centers is concerned, here also the sides of the decagon, tridecile, and dodecagon stand in the middle position, and they are central in this class, between the side of the triangle which precedes and the sides of the pentagon and the pentagonal star which are the central figures in the following class. For it was demonstrated in Book I that in knowledge the dodecagon's side takes precedence over the pentagon's, the tridecile's over the pentagonal star's. Thus knowability leads in the same direction as congruence too, by Proposition VII. This had to be stated in advance, chiefly for the sake of this demonstration, in case the decile or tridecile should be given preference over the quintile and biquintile. If, however, someone should wish to dismiss the figure at the center and seek knowability rather in the one at the circumference, no less than congruence, although it must be admitted that on that basis the decile would be preferred to the quintile, and the tridecile to the biquintile, he should nevertheless remember that the parts of congruence are the most important, as we have shown in Proposition IV. Therefore, it is a greater thing, and has more effect on influentiality, to create a solid figure (which is like a sort of mathematical idea of physical influentiality) than to have a side which is knowable in a more perfect degree. The side of the dodecagon indeed on this basis brings its aspect into the same class as the chords subtended by a tenth part of a circle, and by three tenths, because they contend with each other for pre-eminence in knowability. For just as those two chords are associated with each other, and the smaller is part of the greater in the divine proportion of the extreme and mean, so also the side of the dodecagon and the side of its star are associated, and that in respect of a division and combination, though not proportional. And in fact this latter pair falls into the first kind of inexpressibles, which embraces the binomials and apotomes; but on the other hand the former pair acquires a new property of division according to extreme and mean, as may be seen in Book I. Hence these degrees are not just balanced, but the side of the decagon even has a little the better of it. It was therefore correctly done that I placed the aspect of the quincunx, or 150, in the same degree as the quintile, 72, and the biquintile, 144, though giving the chief seat to these last.

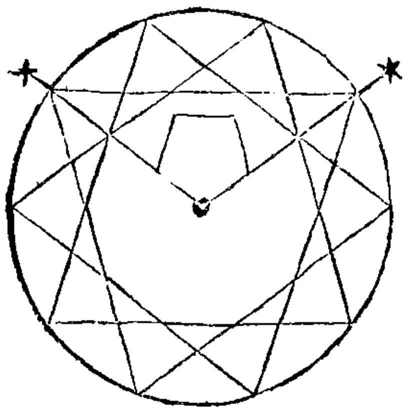
Proposition XIV

The fifth, lowest, and feeblest degree of the aspects is that of the decile and the tridecile, the octile, and the trioctile.

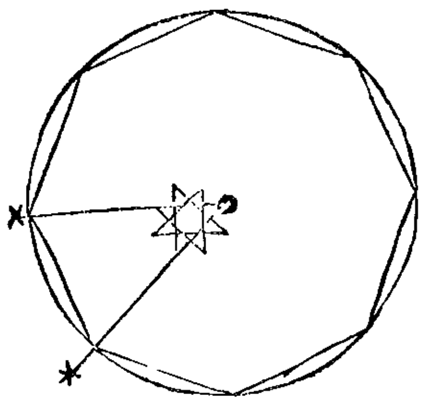
I have made the fifth place for the decile and the tridecile (Maestlin calls them the half-quintile and the quintile -and-a-half) which until now I left out in the Ephemerides. With them I have associated the octile and trioctile, or half-quadrant and quadrant-and-a-half, which the compilers of almanacs, at my suggestion indeed and to a large extent on the authority of Ptolemy, have seized on, but with too much enthusiasm and too little consideration. Therefore, both points must be tested, first that these four are feebler than the quintile and biquintile, and second that the decile and tridecile are stronger than the octile and trioctile, by a very little.



Since, then, our propositions place the greatest weight for determining influence on the congruence of the chief figure, that is the one at the circumference, it is evident *that the pentagon and its star are congruent each with the figures of its own kind, to form a perfect solid, as has just been stated, and are also splendidly congruent with each other to lay out a plane. On the other hand, the decagon and the octagon along with their stars cannot be congruent in the solid, each with the others of its own kind. Indeed, the decagon and the octagon are congruent, but with others which are not all of their own kind. The stars in fact start on a certain congruence in the solid but do not complete it. Even in the plane their congruence is more ignoble, because they do not deliver shared operations, each independently with its own star, as the pentagon does with its own; but with those stars of their own, and the octagon with the tetragon, they come into partnership in an alien congruence, and the decagon*



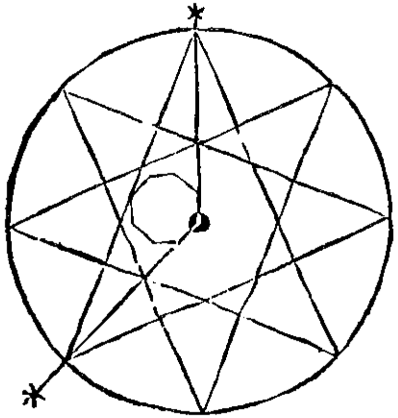
itself prevents the possibility of that's being continued. Its star also produces a split congruence in the intermediate spaces which are intercepted. However, the octagon and its star rejoice in an alternating continuation of congruence, with an admixture of squares. The congruence occurs in a variety of forms. Thus these four are almost on a par in plane congruence, especially as each set of figures has inexpressible areas. But in knowability the pentagonal group is greatly pre-eminent. First, if we were to consider the figures at the center, which in this case are in fact the pentagon and its star, their sides indeed fall under the same class of inexpressibles as the sides of the octagon and its star, being elasson and mizon.[62] *But if we consider the figures at the circumferences, which in this case are the sides of the decagon and its star, these are not only from the nobler kind of binomials and apotomes, whereas the octagon's lines are from the fourth kind, which is that of the mizons and elassons, but also all the sides of the pentagonal group acquire the noblest property of division in the proportion of extreme and mean, which clearly does not apply at all to the octagon's lines. But if the octagonal group seemed to be somewhat pre-eminent in congruence, in this case on the other hand it is much more strongly pushed down by the pentagonal group. I was therefore right to*



[62] See Book I, definition 25.

relegate both sets, as contenders for superiority, to a single class, though putting the pentagonal group in front. Book I should be continually consulted on these points.

It is also a peculiar privilege of the biquintile, in front of the tridecile and the trioctile, and also the quincunx, that the pentagonal star, that is its primary figure, also has an angle which is a very fitting imitator of that of a triangle, because both the three angles of a triangle and also the five angles of the pentagonal star, when combined in each case, equal two right angles. Hence in this way the sides which form the angles each intercept their own arcs of the circle, but not some portion of the circle as pair and pair in common. That cannot happen in the star of eight, ten, or twelve rays, because of their restriction to an even number, and in the other figures of primary origin, because of the size of their angle.



I have therefore discarded all the minor points, to create confidence in the appropriate arguments, in case the number of aspects should become too great, and give birth to confusion in testing. I have therefore to call a halt at the square and the quintile and the biquintile, and not go on to their four derivatives of the last degree, inasmuch as they are very feeble, although the first Proposition admits these as well. If, however, these are not found satisfactory, and if the importance of the first Proposition is so great that all protest against it is prohibited, come, let anyone be free as far as I am concerned to have regard to these aspects also, especially at times when all the rest are absent. For it is right that the testimony of experience should be heard about these as well, seeing that it created the original confidence in the others, before any reasoning.

Proposition XV

There are some configurations which are on the borderline between influential and non-influential, namely the arc of 24° from the pentekaidecagon (fifteen-sided figure), and 18° from the icosagon. **Book I, Proposition XLIV, and Book II, Conclusion XXIX.**

For the figures are knowable, but the former improperly, and the latter in a remote degree, as was demonstrated in Book I.[63] *They are also congruent, but the former not in all angles equally, in the same form of congruence; and the latter in all its angles indeed, but with a congruence which is clearly not continuable, which was shown in Book II. Therefore, there is a certain germ of influentiality, and an attempt, as it were; but the effect is either none or imperfect.*

[63] See Book I, propositions 43 and 44.

These figures have many stars, the former in fact five,[64] the sides of which subtend the following arcs of the circle: 48°, 96°, 112°, 156°, 168°. The latter however has four, the sides of which subtend arcs of the circle of 54°, 126°, 162°, and 171°. But as these stars have re-entrant angles which do not take in other congruent angles (in the way in which the re-entrant angles of the pentagonal and decagonal star take in the angles of the pentagon, those of the octagonal star the angle of the square, and those of the dodecagonal star the angle of the triangle), there remains for them therefore only congruence of the acute angles, which we call the rays. They are therefore more humble than their own figures of primary origin.

[64] Kepler's statements contain errors. The 15-sided polygon has only 3 stars, the corresponding central angles being 48°, 96°, and 168°. The central angles 132° (not 112°) and 156° are the central angles of stars belonging to the 30-sided polygon. Again the 20-sided polygon has only 3 stars. The single 171° is the central angle of a star belonging to the 40-sided polygon.