Claudius Ptolemy's Mathematical System of the World

Waseda University, SILS,
Introduction to History and Philosophy of Science

Roman Egypt, 117 CE

A *mathematical* worldview

In the 2nd century CE, Claudius Ptolemy drew on the ideas and technical devices of his predecessors to create the first complete mathematical description of the whole cosmos (mathematical astronomy, geography and cosmography).

Ptolemy took his technical success as a strong confirmation proof that the world was actually as his models described.

Celestial phenomena: The fixed stars

If we regard the earth as stationary at the center of a spherical cosmos, as Aristotle claimed, we notice a number phenomena that can be explained by this assumption:

The fixed stars move $360°$ each day on circles that are parallel to the equator. (Over long periods of time they also move parallel to the ecliptic – a phenomenon known as procession. We will ignore this for now.)

Latitude of Tokyo, looking north

Celestial phenomena: The seasons

The length of daylight changes throughout the year, and the timing depends on the North-South location.

The sun rises and sets at true East and true West only on the equinoxes. It rises towards the North-East in the summer, and the South-East in the winter. It sets towards the North-West in the summer, and the South-West in the winter.

The seasons are not the same length.

Latitude of Tokyo: Sunset, vernal equinox

Latitude of Tokyo: Sunset, summer solstice

Celestial phenomena: The moon

The moon has phases. It cycles through its phases in about 29 days.

The moon can eclipse the sun and can be eclipsed. This phenomena is related to the phases. (We call this *synodic* phenomena because it is linked to the sun – *synodos* is a Greek word meaning "meeting," "assembly," "coming together.")

The moon returns to the same *sidereal position*[1] in about 27 days.

(The moon also has a period relative to the plane of the zodiac, which we will ignore for now.)

[1] Position relative to the stars.

Celestial phenomena: The planets

Against the background of the *fixed stars*,[2] all of the planets periodically appear to stop (*station*) turn around and go the opposite direction for a while (*retrogradation*).

Stations and retrogradations are related to angular distance from the sun. (We call this the *synodic* anomaly.)

- The inner planets (Venus and Mercury) are always within a certain angular distance from the sun.
- The outer planets (Mars, Jupiter and Saturn) can be any angular distance from the sun.

The sizes and distribution of the retrograde loops is related to their position relative to the zodiac. (We call this the *zodiacal* anomaly.)

[2] The stars are not actually fixed, but they appeared to be fixed relative to one another.

Mars: Retrograde loop

Mars: 7 years, no image of earth, no daylight

Predictive Greek Astronomy

Apollonius of Perga (c. 263–190 BCE).

- Most famous as a mathematician – theory of conic sections, geometric analysis, etc.)
- Devised, or at least used, the *epicycle* and *eccentric* models.
- We do not know if his astronomy was predictive or descriptive.

Hipparchus of Nicaea (c. 150–127 BCE).

- Influenced by Babylonian astronomy, produced the first predictive astronomical works in Greek.
- Developed predictive models using the *epicycle* and *eccentric* models.
- Ptolemy took many of his ideas and observations from Hipparchus.

Worked at Alexandria (or Canopus, a suburb of Alexandria) with observations dated 127–141 CE.

Produced crucially important works in every field in which he was active.

Works: Minor works (mostly astronomical), *Harmonics* (music theory), ***Almagest***, *Tetrabiblos* (astrology), ***Planetary Hypotheses***, ***Geography***, *Optics*.

Original title: *Systematic Mathematical Treatise*.

Hypotheses: (1) A stationary spherical earth in the center of a spherical cosmos. The earth is tiny relative to the size of the cosmos. (2) Celestial bodies move about the earth with two motions: (2a) *diurnal motion*[3] to the west, and (2b) *proper motion* to the east.

13 Books (like Euclid's *Elements*), relatively few observations, ancient trigonometry, mathematical deduction.

[3] That is, daily motion

Goal & structure of the *Almagest*

Geometric models of all of the celestial bodies (eccentric, epicycle and equant).

At any place on earth, at any time, to predict the positions, and synodic phenomena, of the celestial bodies (stars, Sun, Moon, Mercury, Venus, Mars, Jupiter, Saturn).

Order of topics: Mathematical preliminaries, spherical astronomy, solar theory, lunar theory, eclipse theory, the star catalogue, planetary longitudes, stations and retrogrations, planetary latitudes.

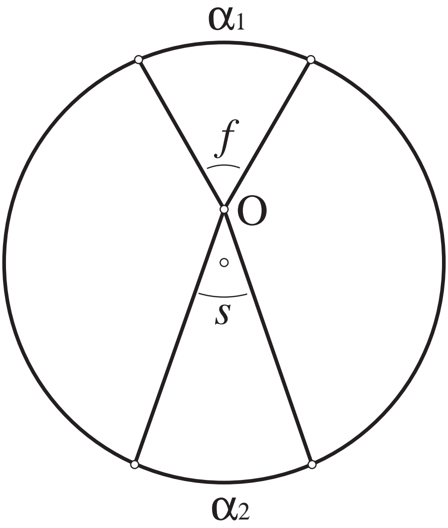
Mathematics & Models

Ptolemy uses geometric models to describe the motion of the heavenly bodies

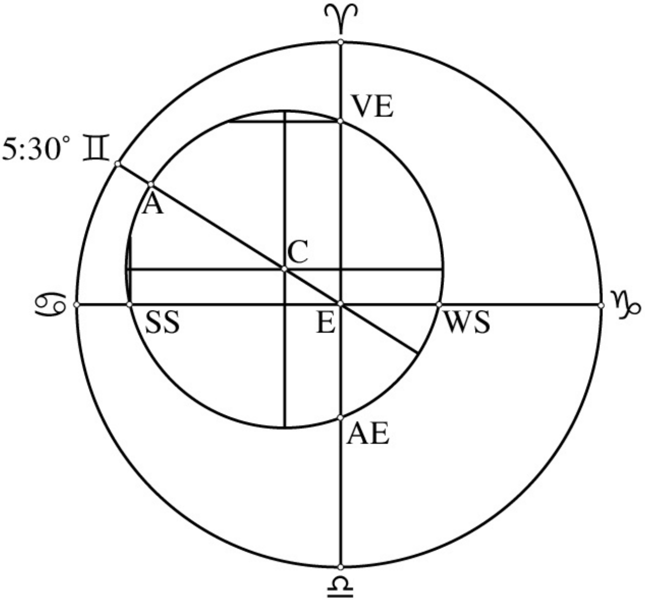
- Eccenter
- Epicycle
- Equant

He uses observations to input numerical values and then uses chord-table trigonometry to compute the details of the models.

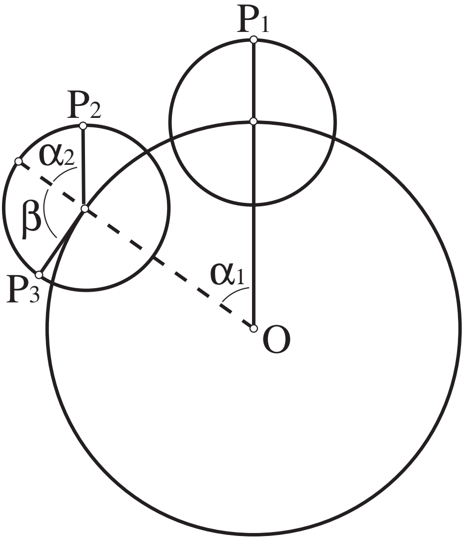
Eccentric Model



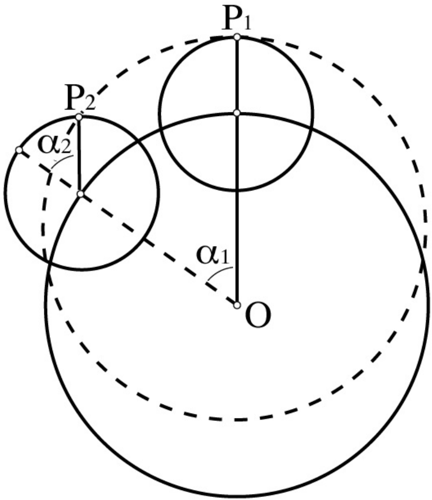
Ancient Solar Model (due to Hipparchus)



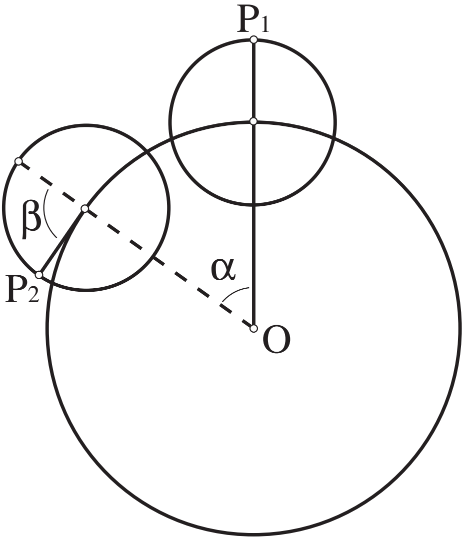
Epicycle Model, I



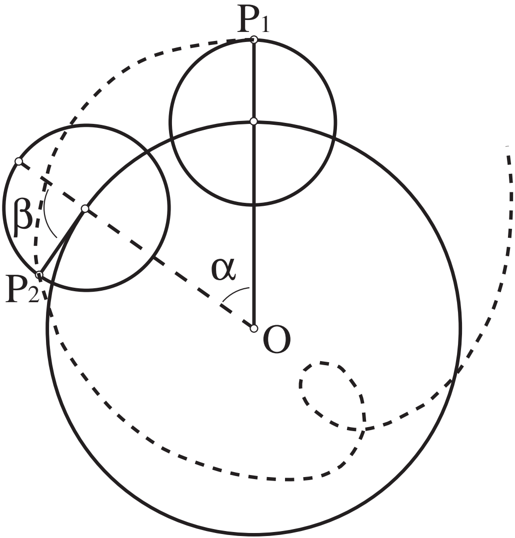
Epicycle Model, II

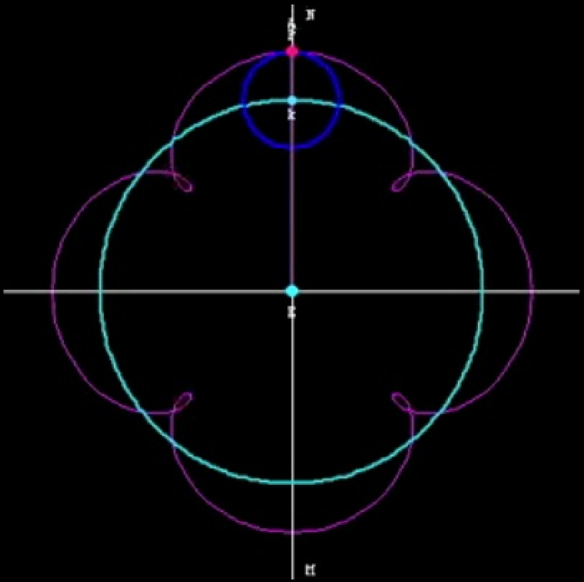


Epicycle Model, III



Epicycle Model, IV





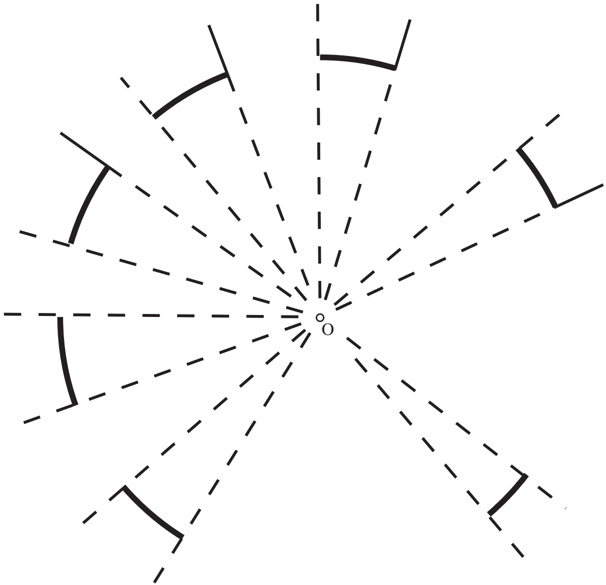
The Equant Model

- The equant model may have been devised by Ptolemy or perhaps was devised by an unknown astronomer sometime between Hipparchus and Ptolemy.
- The goal of the equant is to account for the *zodiacal*, as well as the *synodic anomaly*.

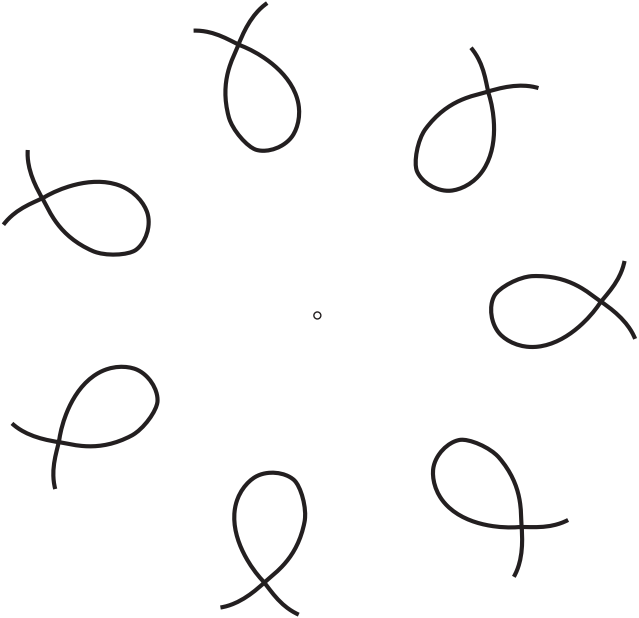
Synodic Anomaly: the phenomena of stations, retrogradations etc., are related to the solar position.

Zodiacal Anomaly: the size and distribution of the retrograde arcs depends on their longitude – that is, position in the zodiac.

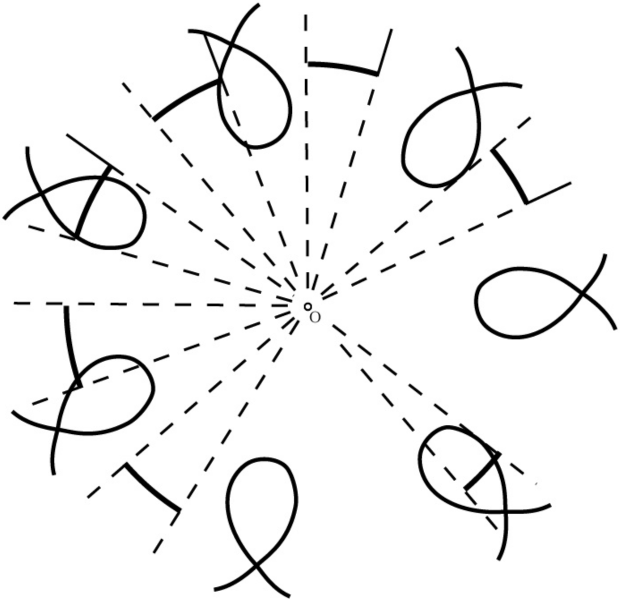
Observed Loops of Mars (about 13 years)



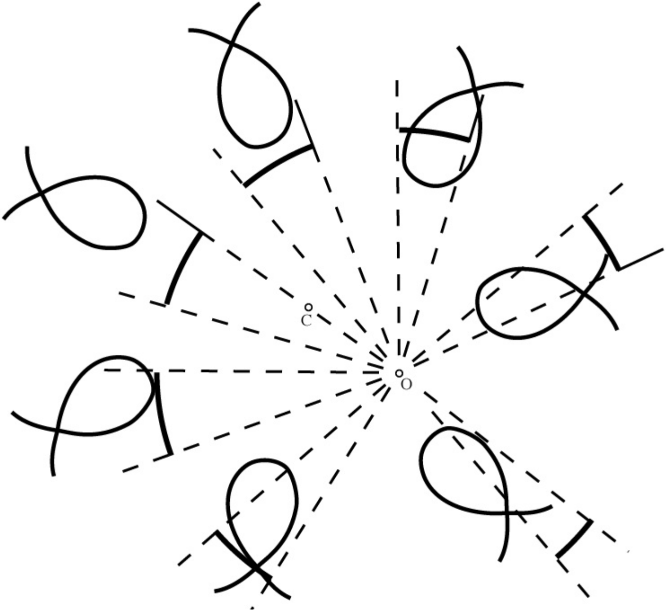
Loops with Simple Epicycle



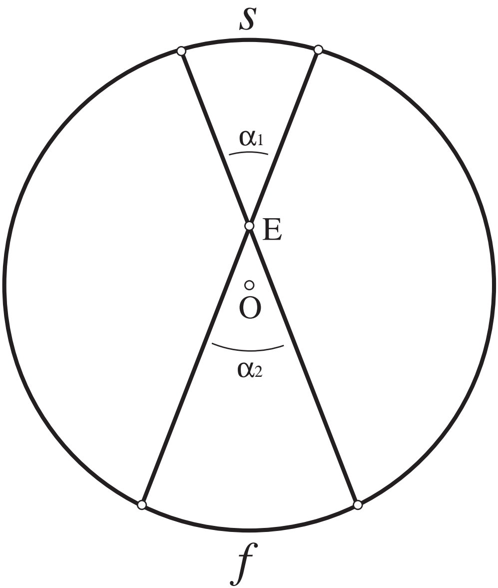
Loops with Simple Epicycle

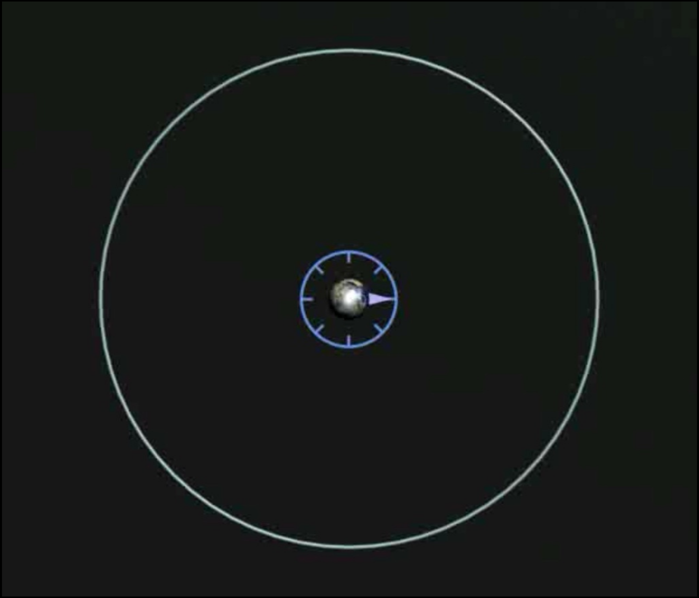


Loops with Epicycle and Eccentric

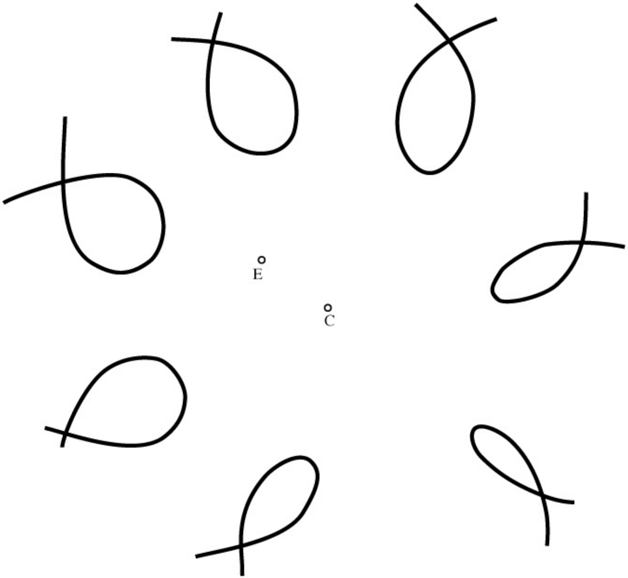


The Equant Model

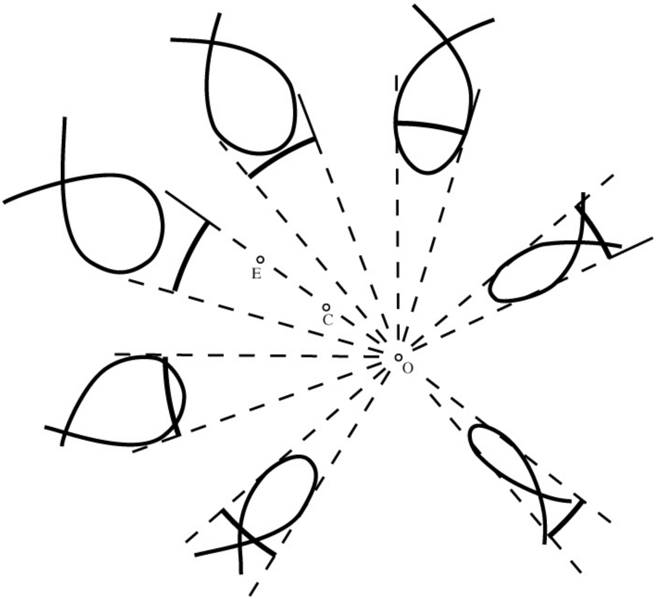




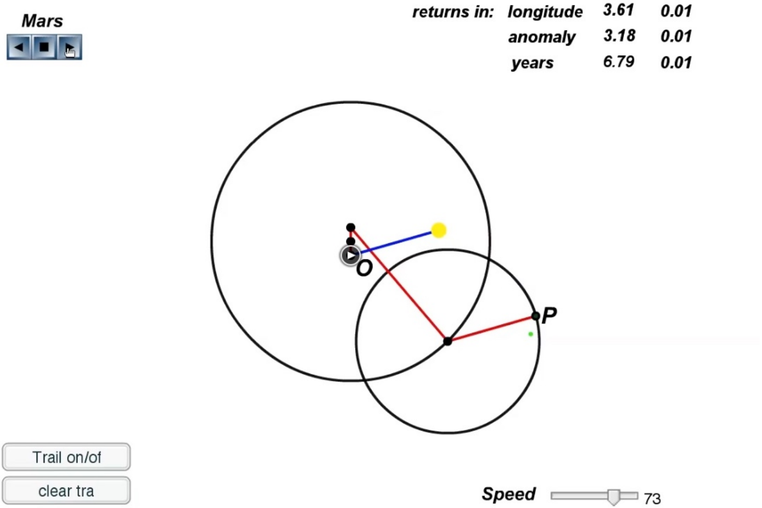
Loops with Epicycle and Equant



Ptolemy's Full Model



Ptolemy's Model of Mars



Mars from a “Geocentric” view

Problems with the *Almagest*

There are, in fact, a number of problems (anomalies) with the observational evidence used in the *Almagest*.

Here are just two examples:

- Solar Year: Two vernal and two autumnal equinoxes "measured" – all off by a day in the same direction but agree exactly with Hipparchus's slightly incorrect year length.
- Greatest Elongations of Venus ($\approx 45°$–$47°$): Two "observed" greatest elongations just over a month apart whereas according to theory (ours and his) these occur just over one and a half years apart.

The *Planetary Hypotheses*

An account, in two chapters or books, of the *physical structure* of the cosmos.

Planetary Hypotheses describes physical models of nested spheres, which show the actual motions of the planets (latitude and longitude).

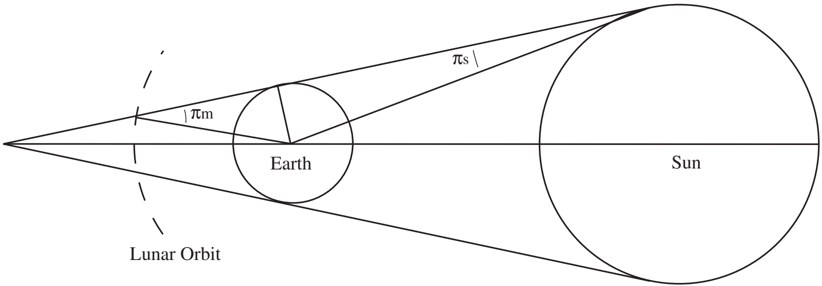
These models are also meant to be of use to instrument makers who want to build physical replicas of the cosmos.

Planetary Hypotheses contains calculations of the *absolute* sizes and distances of all the celestial bodies.

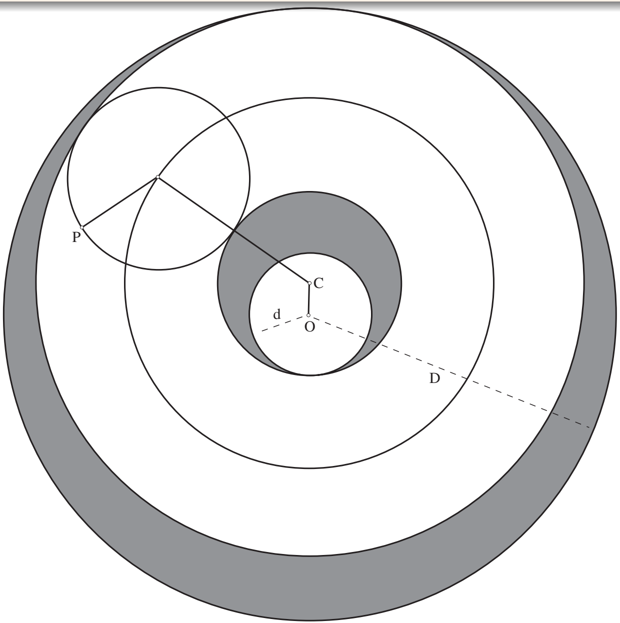
The Logical Structure of the *Planetary Hypotheses*

- The *Almagest* gives the ratios for greatest and least distances of all the celestial bodies $(D : d)$.
- Parallax and angular diameter measurements are used to get *absolute distances* for the sun and moon (in terms of the earth radius, **er**).
- The rest of the celestial distances are calculated on the basis of the sun and the moon.
- The sizes are calculated on the basis of the distances and *assumed* angular diameters.
- This is all fairly rough, but it leads Ptolemy to be able to state absolute sizes and distances.

The Eclipse Diagram



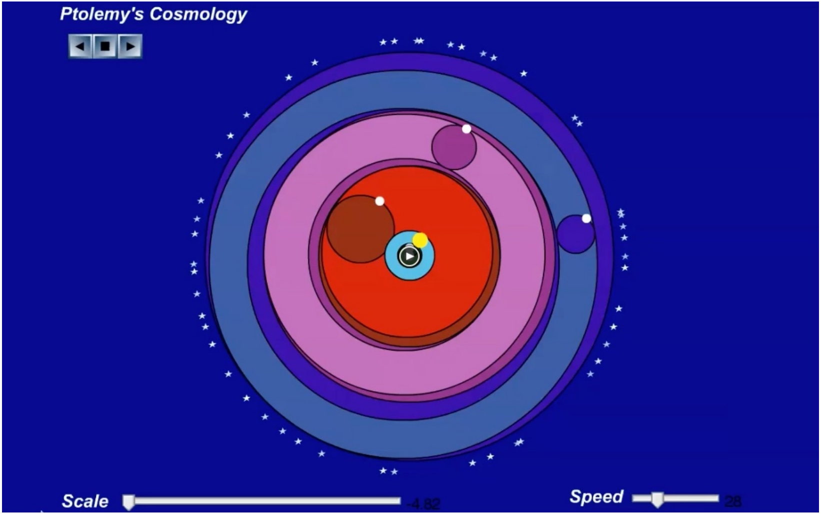
Nested Spheres



The Sizes and Distances

Body	Mean Distance (er)	Size (ev)
Moon	48	1/40
Mercury	115	1/19,683
Venus	622 1/2	1/44
Sun	1,210	166 1/3
Mars	5,040	1 1/2
Jupiter	11,504	82 1/4+1/20
Saturn	17,026	79 1/2
Stars (1st m)	20,000	94 1/6+1/8

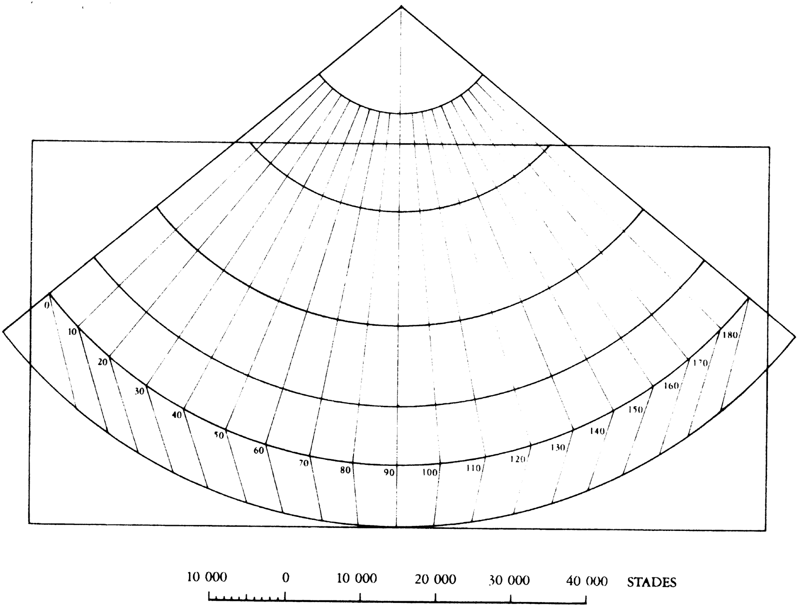
The units are as follows: **er** is "earth radius," and **ev** is "earth volume." Since the Greeks had a fairly decent value for the circumference of the earth, these can be considered absolute units.


Ptolemy's Cosmology
Scale
Speed

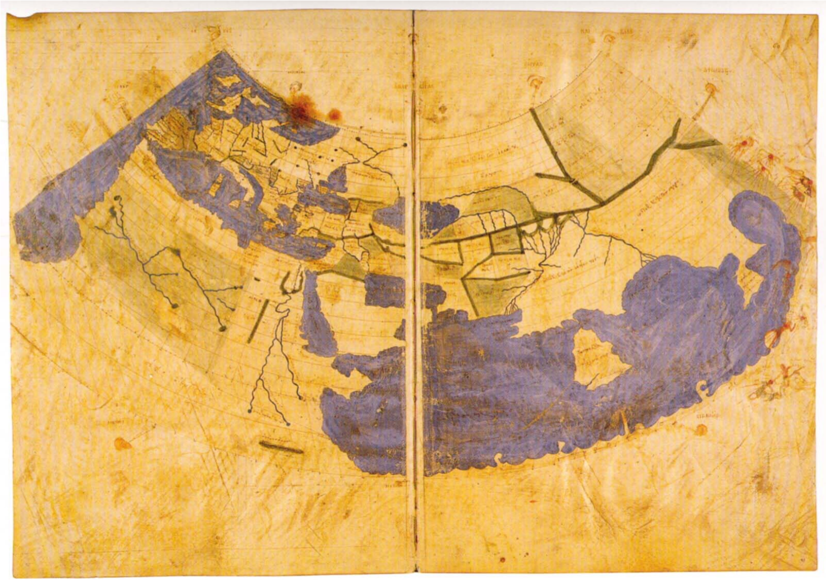
Ptolemy's *Geography*

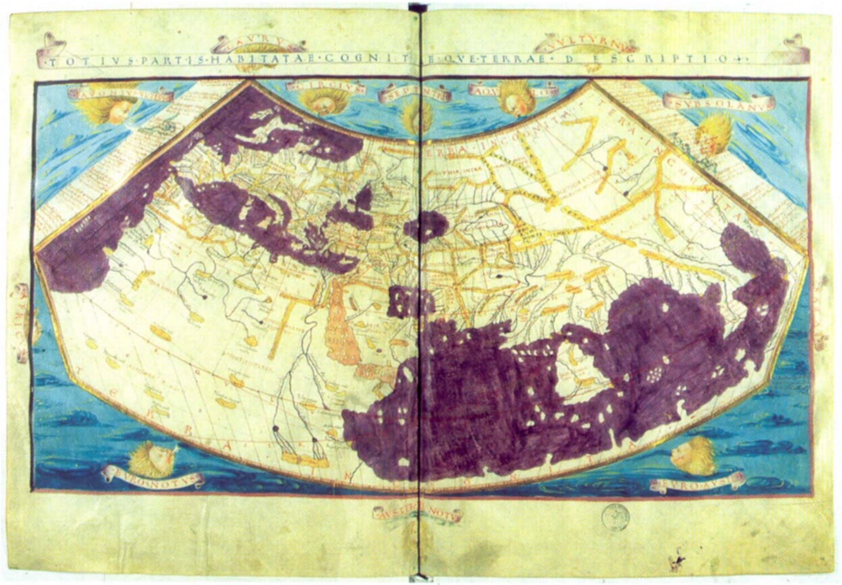
- 8 books of pure *mathematical cartography*.
- The theoretical sections discuss the aim of cartography, the auxiliary role of astronomy, types of projection, criticisms of predecessors' work, analyses of travel reports, etc.
- The bulk of the text consists of lists of coordinate points for topographical features (coast lines, rivers, mountains, etc.), ethnic and political regions, and cities.

Ptolemy's First World Map Projection



Ptolemy's First World Map, Urb. Gr. 82 (15th century)

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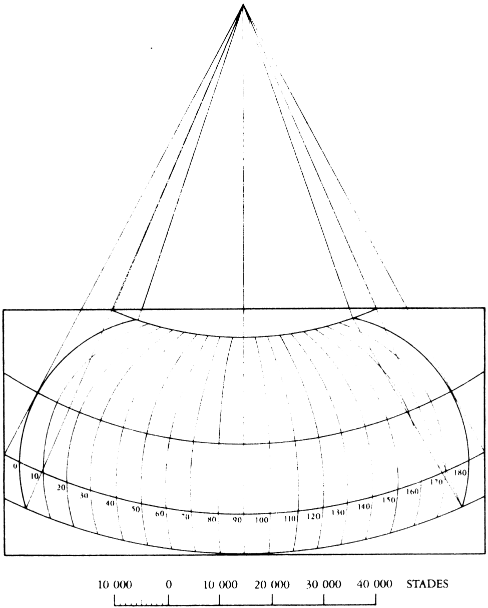
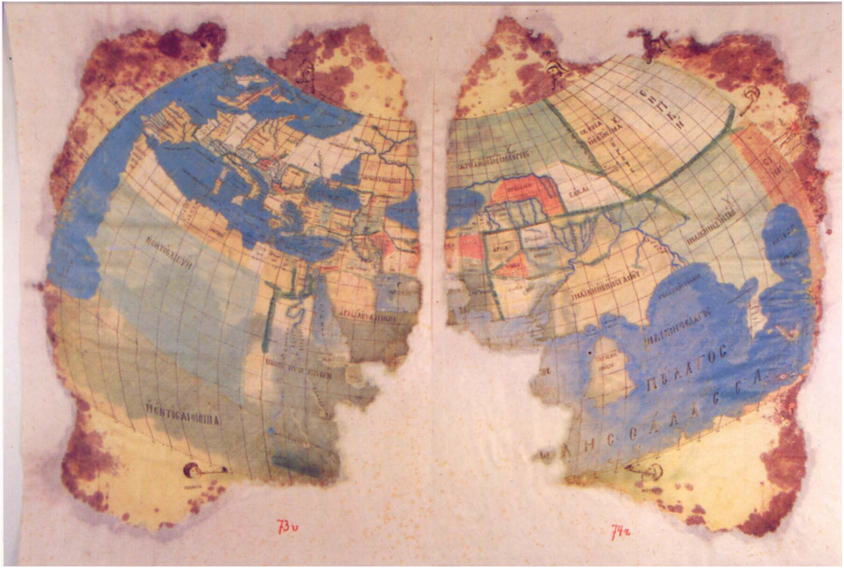
SVBSOLANVS
EVRONOTVS
TERRA INCOGNITA

Ptolemy's Second World Map Projection



Ptolemy’s Second World Map, Constan. Sergag. 57

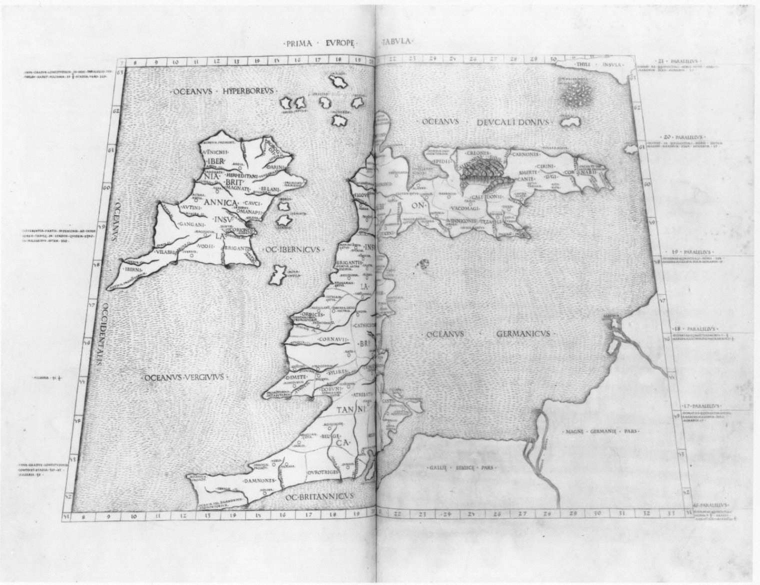


Geography, Ulm Latin edition, 1482

Detailed Europe Map, (15th century manuscript)

Regional Map, Source of the Nile

Regional Map



Overview

- Ptolemy was an expert at developing *mathematical models* to describe the phenomena.
- There is a lengthy process of model fitting, which we have largely ignored. This is what consumes most of the time and energy.
- Ptolemy took the fit between his elaborate models and the phenomena as a strong confirmation argument that his models were correct.

- Should we understand Ptolemy as predominantly an instrumentalist or a realist?