Waseda University, SILS, History of Mathematics



Domenico Fetti, "Archimedes," 1620.

L_Outline

Introduction

Archimedes' Works

Measurement of the circle

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Ancient cultures around the Mediterranean



Mesopotamia (3000 B.C. - A.D. 100)

Egypt (3000 B.C. - A.D. 300)

Greek states (1000 B.C. - 330 B.C.)

Hellenistic kingdoms (330 B.C. - 30 B.C.)

Roman Empire (30 B.C. - A.D. 400)

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Ancient Italy and Sicily, around 200 BCE



What we know about Archimedes (c. 287-212)

- He himself tells us, in various writings, that his father was an astronomer and one of his short works is addressed to the young prince of Syracuse (now in Sicily). He is reported to be related to king Hiero II (of Syracuse).
- He apparently lived and worked in Syracuse and corresponded with other mathematicians, mostly in Alexandria.
- A number of his mechanical writings are preserved in Doric Greek, the dialect of ancient Sicily.
- There are a number of reports that Archimedes was killed, in his 70s, when the Romans conquered Syracuse, 212 BCE.
- There are many legends about him, but it is difficult to sift the truth from the fantasy.

The legend of Archimedes, 1

- Already in antiquity, Archimedes was seen as the iconographic image of the brilliant scientist (like an ancient Einstein).
- Archimedes is said to have been a gifted engineer as well as a mathematician, however, few of his mechanical works survive.
- He is reported to have invented a planetarium that could model, or predict, the positions of the sun, moon and planets.
- There is the famous story of him discovering the principle of density (specific weight) while taking a bath ("Eureka!").

The legend of Archimedes, 2

- The most famous legends circulate around his death.
- When the Romans besieged Syracuse, they are said to have been held off for months by the various war machines invented by Archimedes, such as a "claw" (for capsizing ships) and a "burning mirror" (for igniting ships).
- Although the Roman general Marcellus gave orders that Archimedes was to be captured alive, he was somehow killed when the city was sacked ...

The sources for his writings

- ► A fair number of his writings survive in *early modern* Greek copies (14th-15th century CE).
- All of the complete medieval copies of his works have been lost.
- But, there is one medieval manuscript of his writings in a palimpsest, known as the "Archimedes Palimpsest."¹
- There are also highly literal translations of his works made from Greek into Latin by William of Moerbeke (c. 1215–1286).

¹A palimpsest is a text made by cleaning and reusing an older text.

The Archimedes Palimpsest

- In 1906, the Danish philologist Johan Ludvig Heiberg discovered a text of Archimedes in a medieval palimpsest in Constantinople (now Istanbul).
- ► This was then "lost" during WWI.
- In 1998, it was sold to an anonymous buyer for \$2 million USD, at Chirstie's Auction House, London.
- It is now held at the Walters Art Museum in Baltimore and has been studied by scholars.
- It contains (among others) the following texts by Archimedes: Equilibrium of Planes, Spiral Lines, Measurement of a Circle, On the Sphere and Cylinder, On Floating Bodies (only copy in Greek), Stomachion (only copy), The Method of Mechanical Theorems (only copy).

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Facing pages of the Christian prayer book text, in codex format.

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- A full leaf of the palimpsest, turned on its side to show the original Archimedes columns, in roll format.
- We see two columns of Archimedes' text, with some diagrams.
- This image has been modified, using modern technology, to bring out Archimedes' text.



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- An image of the same leaf made using modern imaging technologies (x-ray fluorescence from the Stanford Linear Accelerator).
- The physical spectrum of the underlying ink has been exploited to make Archimedes' text even clearer.



Scenes from a Nova documentary...

Archimedes' Works

Archimedes' mathematical style

- Archimedes developed a different approach to writing mathematical texts than that of Euclid.
- Instead of focusing on extended texts of (often simple) theory, he tried to solve difficult problems and developed the necessary theoretical tools along the way. His texts were not, however, any more "practical" than Euclid's.
- In particular, he was interested in how to make measurements of curvilinear shapes.
- We can characterize Archimedes as developing a *mathematics of measuration*.
- He also worked on various aspects of mechanics, such as locating centers of gravity of bodies in the air and in fluids, the tipping points of floating bodies and the relationship between volume and density.

Archimedes' Works

Archimedes' works

- Works on measurement: On Spirals (28 props.), On Conoids and Spheroids (32 props.), On the Sphere and the Cylinder (2 books), The Quadrature of the Parabola (24 props.), On the Measurement of a Circle (3 props.)
- Works on mechanics: On the Equilibrium of Planes (2 books), On Floating Bodies (2 books),
- Others: Various riddles, mathematical puzzles and slights of hand. The *Method* (a treatise showing how to use the idea of infinitesimal lines and planes to calculate areas and volumes, by means of a "virtual" balance).

Archimedes' Works



The blue sphere has both $^{2}/_{3}$ the surface area and $^{2}/_{3}$ the volume of the gray cylinder. (From: *On the Sphere and the Cylinder*.)

Measurement of the circle

Archimedes' On the Measurement of a Circle

- Preliminaries: (1) the circumferences of circles are proportional to the diameters (^{C₁}/_{C₂} = ^{d₁}/_{d₂}), (2) *Elem.* XII 2 the areas of circles are as the squares on the diameters (^{A(C₁)}/_{A(C₂)} = ^{d²}/_{d²}), (3) the area of a regular polygon is ¹/2hQ where *h* is the apothem and *Q* is the perimeter, (4) the difference between the area of a circle and an inscribed (or circumscribed) polygon can be made less than any area.
- Prop. 1: The area of a circle is equal to a right angled triangle in which one side is equal to the radius and the other is equal to the circumference. A proof using the method of exhaustion and a double indirect argument.
- Prop. 3: The ratio of the circumference of a circle to its diameter, π, is greater than 3¹⁰/71 ≈ 3.14089... and less than 3¹/7 ≈ 3.14285...

Measurement of the circle

Archimedes' mathematics

- Archimedes pioneered the techniques of measuring areas and volumes using techniques of pre-calculus that would be not be taken up again until the medieval Islamic, and the early modern periods.
- He developed the practice of solving difficult problems using a *more advanced set of assumptions* than those of the *Elements*.
- He laid the foundations of the mathematical theory of statics - the mechanics of systems of bodies in equilibrium.
- His mathematical techniques were extended by only a few mathematicians working in Arabic (there was not much transmission in Latin) during the middle ages and were not fully developed before the early modern period.