

Viète and Descartes

Symbolic Analytic Geometry and the Origins of Symbolic Algebra

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History of Mathematics

Introduction

François Viète

Life and work

Renè Descartes

Life and work

Descartes' mathematical approach

The locus problem

The early modern period in Europe

- ▶ The early modern period runs from the Renaissance to around 1700.
- ▶ It was characterized by various overall patterns such as:
 - ▶ *globalization*: exploration, colonization, more rapid forms of transportation, communication, financial transactions, etc.
 - ▶ *secularization*: the Reformation, separation of church and state, the rise of national monarchies, etc.
 - ▶ *the birth of capitalism*: a rise of the mercantile and middle classes, division of labor, etc.
 - ▶ *the scientific revolution*: development of new philosophies and technologies, production of new knowledge about the physical world, increased access to scientific activities by the middle class, etc.
- ▶ Some major historical events: the Reformation, Thirty Years War, the commercial revolution with international trade, European colonization, development of the calculus...

Intellectual context of early modern mathematics

- ▶ The early modern period saw an increasing dissatisfaction with received forms of knowledge and established institutions.
- ▶ Aside from university professors and astronomers, there were still *almost no jobs* for professional mathematicians, hence many worked outside of the context of the university system, and produced mathematics as a leisure activity in their free time.
- ▶ Mathematicians of this time tried to solve problems that the ancients could not solve and to do so, they developed entirely new fields of mathematical approach — symbolic algebra, analytical geometry, probability theory, symbolic number theory, the calculus...

François Viète (1540-1603)

- ▶ Viète came from a legal family and himself served as privy council to Henry III and IV, of France.
- ▶ He also made himself valuable to the king by using mathematics to break various Spanish codes used to encrypt diplomatic messages.
- ▶ He is most known for having developed a new algebra on a new logical foundation.



Viète's contribution

- ▶ Viète introduced the idea of using symbolic algebra to solve *geometric* problems.
- ▶ He sought to place algebra on a rigorous foundation by basing the algebraic operations on fundamental arithmetic (addition, subtraction, multiplication, division, etc.).¹
- ▶ He transformed the focus of algebra from solving individual numerical problems, to furnishing *general patterns of solution*, which could be understood and exhibited symbolically.
- ▶ This paved the way for making symbolic equations themselves objects of mathematical study...

¹ This may seem obvious, but is not the only way to conceive of algebraic operations. Viète did not originate this practice, but followed an old tradition developed by medieval Islamic mathematicians.

Renè Descartes (1596-1650)

- ▶ Born to a distinguished legal family—he was a great disappointment to his father.
- ▶ After taking a license in law, he worked in his youth as a gentleman officer, a mercenary.
- ▶ He lived off of his estate and slept in late every day.
- ▶ Died of pneumonia in Stockholm (tutor to Queen Cristina).
- ▶ His books were placed on the index of banned books in 1663.



Descartes' *Rules for the Direction of the Mind*, 1

Rules for the Direction of the Mind

“The first rule was never to accept anything as true unless I recognized it to be evidently such: that is, carefully to avoid precipitation and prejudgement, and to include nothing in my conclusions unless it presented itself so clearly and distinctly to my mind that there was no occasion to doubt it.

The second rule was to divide each of the difficulties which I encountered into as many parts as possible; and as might be required for an easier solution.

Descartes' *Rules for the Direction of the Mind*, 2

The third was to think in an orderly fashion, beginning with the things which were simplest and easiest to understand, and gradually and by degrees reaching toward more complex knowledge, even treating as though ordered materials which were not necessarily so.

The last was always to make enumerations so complete, and reviews so general, that I would be certain that nothing was omitted."

That is: (1) nothing is to be assented to unless self-evident, (2) every topic is to be divided into the smallest possible parts, (3) each part should be considered in the right order, the simplest first, and (4) no part is to be omitted in discussing the whole.

Descartes' symbolic approach

- ▶ Descartes' most important mathematical work, the *Geometry* (1637), was one of three appendixes to a philosophical text, *Discourse on Method*.
- ▶ He begins by showing how to make simple geometric constructions so that we can interpret all the arithmetic operations ($x + y$, $x - y$, xy , $\frac{x}{y}$, x^2 , \sqrt{x}) as producing **lines**.
- ▶ In this way, any geometric problem can be transformed into an algebraic problem.
- ▶ Descartes introduced the system of using letters early in the alphabet (a , b , c) to indicate **values that are known** and fixed from the beginning and letters late in the alphabet (x , y , z) to indicate **values that are unknown** or vary.

Descartes' theory of equations

- ▶ Descartes wrote out **equations**, such as $ax^2 + bx + c = 0$, and used formal considerations to study the equations themselves.
- ▶ This led to a **theory of equations** that described what kinds of equations are related to what kinds of geometrical curves, and what kinds of solutions they have.
- ▶ For example:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

is the general equation for a *conic section*, where a circle is $(x)^2 + (y)^2 = r^2$, a parabola is $4px = y^2$, an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and a hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

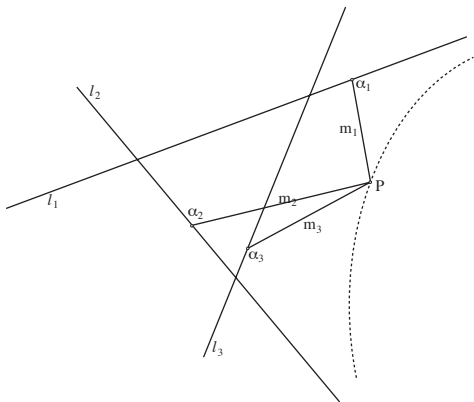
Ancient geometric analysis

- ▶ The Greek mathematicians had developed a style of problem-solving known as geometrical **analysis**, in which the mathematician *assumes* what is to be found and used this assumption to demonstrate something simpler.
- ▶ As an example, we look at Pappus, *Collection* VI.39-40, to trisect a **given** angle.²
 - ▶ We begin by assuming that the angle ABC has *already* been trisected.
 - ▶ We then show that $DE = 2AB$, hence, if we can construct the line DE to meet this condition, we will have solved the original problem...

² Here, *given* means something like "fixed and known."

The three (and four) line locus problem

- ▶ We suppose three lines, l_1, l_2, l_3 , given in position,³ and three other lines, m_1, m_2, m_3 , which make given angles, $\alpha_1, \alpha_2, \alpha_3$, with the given lines.
- ▶ What is the **locus**⁴ of point P such that:
 3 lines: $m_1 m_2 = l m_3^2$, or
 4 lines: $m_1 m_2 = k m_3 m_4$?

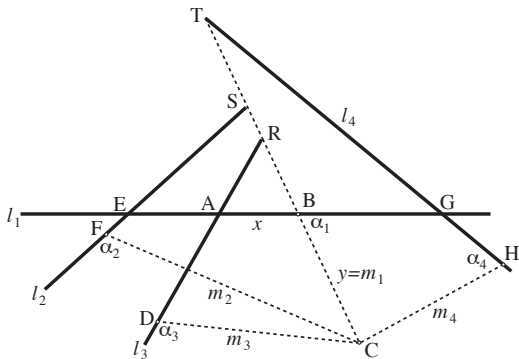


³ They don't move around, and we know where they are.

⁴ A locus is the set of all points (usually forming a curve or surface) satisfying some condition.

Solution of the four line locus problem

- ▶ To find m_1, m_2, m_3, m_4 in terms of two unknown variables, x and y , and some set of given lengths, a, b, c , etc.
- ▶ We can then state the general form of the equation $m_1 m_2 = k m_3 m_4$ in terms of x and y .
- ▶ We show that it is an equation of the form $ax^2 + bxy + cy^2 + dx + ey + f = 0$.



Overview

The introduction and systematic use of abstract symbols had a profound effect on the development of mathematics.

- ▶ Mathematicians began to do work that focused on the patterns and objects exhibited in the symbols themselves, as well as in the objects that they had originally been introduced to symbolize.
- ▶ Previously somewhat separated objects of investigation could be grouped together when they could be handled by the same symbols and the operations on them.
- ▶ Symbolic representation allowed one to more easily develop general arguments and apply them to wide classes of problems.