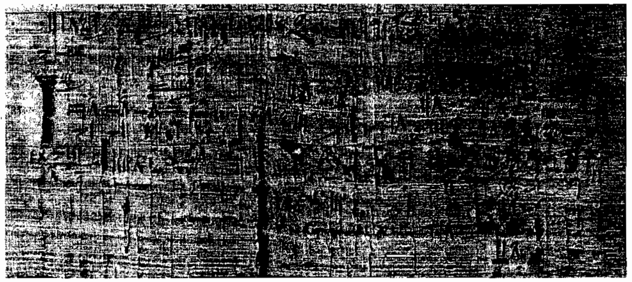

Rhind Mathematical Papyrus Problems 26 and 27.
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Rhind Mathematical Papyrus, Problem 26

A quantity, its $\bar{4}$ (is added) to it *so that 15 results*

Calculate with 4.
You shall calculate its $\bar{4}$ as 1. Total 5.
Divide 15 by 5.
\ . 5
\ 2 10
3 shall result.
Multiply 3 times 4.
. 3
2 6
\ 4 12
12 shall result.
. 12
$\bar{4}$ 3 Total 15.

The quantity 12
its $\bar{4}$ 3, **total 15.**

This problem belongs to the group of ʿḥʿ-problems, named after the characteristic term used in the title of each of these problems. ʿḥʿ is the Egyptian word for "quantity" or "number." The ʿḥʿ-problems, as can be seen from the example above, teach the procedure for determining an unknown quantity (ʿḥʿ) from a given relation with a known result. This example presents a quantity to be determined, which becomes 15 if its fourth is added to it. The text of the problem can be divided into three sections:

- title and given data
- procedure to solve the problem
- verification

The beginning of the problem is marked by the use of red ink (rendered as bold print in the transliteration). The procedure is then given as a sequence of instructions, sometimes followed by their respective calculations. For example, after the instruction "divide 15 by 5" we

see the actual operation carried out. Once the result is obtained a verification is executed, first in the form of a calculation and then indicated by the use of red ink, as a complete statement.

In order to achieve a close reading of the source text, the individual steps of the solution have to be followed as such. We can make this procedure clearer if we rewrite the given instructions using our basic mathematical symbolism (+, –, ×, ÷). The procedure stated in the problem looks as follows after this rewriting ([ ] indicate ellipses in the text):

data		$\bar{4}$
		15
sequence of instructions	**1**	$[1 \div \bar{4}] = 4$
	2	$4 \times \bar{4} = 1$
	3	$4 + 1 = 5$
	4	$15 \div 5 = 3$
	5	$3 \times 4 = 12$
verification	$\mathbf{v_1}$	$12 \times \bar{4} = 3$
	$\mathbf{v_2}$	$12 + 3 = 15$

The text starts by announcing the given data of the problem: $\bar{4}$ and 15. In the rewritten form they are noted above the sequence of instructions. The instructions begin with "Calculate with 4." Since 4 is the inverse of the first datum ($\bar{4}$), there must have been one step in the calculation that has not been noted in the source text, namely the calculation of the inverse of $\bar{4}$. In the rewritten procedure above, we include this as step **1**. To indicate that it was not noted in the source text, we use square brackets ($[1 \div \bar{4}]$). Step **2** is the multiplication of the result of step **1** with the first datum ($4 \times \bar{4}$). Step **3** adds the result of steps **1** and **2**: 4 + 1. Step **4** uses the second datum (15) and the result of step **3**: 15 ÷ 5. Step **5** finally is the multiplication of the results of steps **1** and **4**: 3 × 4.

By following the procedure in this rewritten form several observations can be made. The basic structure of the text is sequential; results obtained in one step may be used in later step(s). Thus the result of **1** is used in **2**, **3**, and **5**; the result of **2** is used in **3**, the result of **3** is used in **4**, and the result of **4** is used in **5**. Data can be used at any time in the procedure. In this example the first datum ($\bar{4}$) appears in steps **1** and **2**; the second datum (15) in step **4**. Other numbers appearing in the instructions are either inherent to the specific mathematical operation carried out (e.g., the number 1 in the calculation of the inverse), or to the procedure itself (we will see an example of this later). The scribe must have known these numbers; they were learned with the sequence of operations of the procedure.

The different categories of "numbers" can be made even more obvious by rewriting the procedure again, this time indicating the data as D_1 ($=\bar{4}$) and D_2 (=15), and the result of step number *n* by ***n***, and the constants as before by their numerical value:

	D_1
	D_2
1	$[1 \div D_1]$
2	$\mathbf{1} \times D_1$
3	$\mathbf{1} + \mathbf{2}$
4	$D_2 \div \mathbf{3}$
5	$\mathbf{4} \times \mathbf{1}$
$\mathbf{v_1}$	$\mathbf{5} \times D_1$
$\mathbf{v_2}$	$\mathbf{5} + \mathbf{v_1} = D_2$

Again the sequential character is obvious. Rewriting procedure texts in this way enables a modern reader to compare the procedure of different problems more easily, as well as to see similarities between individual examples.

The solution of this example uses the so-called method of false position. A wrong solution (= 4) is assumed. In order to make this wrong solution suitable for the following calculations, it is determined here as the inverse of the first datum. The unknown (false solution) and its fractional part are then added (= 5). This is compared to the given (correct) result (= 15). Since the result obtained with the assumed number is three times smaller than the given result, the assumed number has to be multiplied by 3 to obtain the correct solution.

Rhind Mathematical Papyrus, Problem 27

A quantity, its $\bar{5}$ (is added) to it *so that 21 results*

.	5	
$\bar{5}$	1	Total 6.
\ .	6	
\ 2	12	
\ $\bar{2}$	3	Total 21.
\ .	3$\bar{2}$	
2	7	
\ 4	14 (sic! source text 15)	

The quantity 17 $\bar{2}$,

its $\bar{5}$ 3 $\bar{2}$ **Total 21.**

Problem 27 also belongs to the group of *ꜥḥꜥ*-problems. Indeed, it is very similar to its predecessor, problem 26. However, after the title, which again includes the given data, only three calculations are noted, and not a single instruction. A comparison with the calculations of problem 26 reveals that the procedure of solving this problem is identical. This can best be seen if we rewrite the procedure in the same way as we have done in problem 26. The rewritten procedure shows the similarity (operations are reconstructed based on the calculations):

	No. 27				*No. 26*
	$\bar{5}$		D_1		$\bar{4}$
	21		D_2		15
1	$[1 \div \bar{5}] = 5$	1	$[1 \div D_1]$	1	$[1 \div \bar{4}] = 4$
2	$5 \times \bar{5} = 1$	2	$1 \times D_1$	2	$4 \times \bar{4} = 1$
3	$5 + 1 = 6$	3	$1 + 2$	3	$4 + 1 = 5$
4	$21 \div 6 = 3\bar{2}$	4	$D_2 \div 3$	4	$15 \div 5 = 3$
5	$3\bar{2} \times 5 = 17\bar{2}$	5	4×1	5	$3 \times 4 = 12$
v_1	$17\bar{2} \times \bar{5} = 3\bar{2}$	v_1	$5 \times D_1$	v_1	$12 \times \bar{4} = 3$
v_2	$17\bar{2} + 3\bar{2} = 21$	v_2	$5 + v_1 = D_2$	v_2	$12 + 3 = 15$

Moscow Mathematical Papyrus, Problem 25

Method of calculating a quantity calculated times 2
together with (it, i.e., the quantity), it has come to 9.
Which is the quantity that was asked for?
You shall calculate the sum of this quantity and this 2.
3 shall result.
You shall divide 9 by this 3.
3 times shall result.
Look, 3 is that which was asked for.
What has been found by you is correct.

This example from the *Moscow Mathematical Papyrus* shows several differences to the style of the *Rhind Mathematical Papyrus*. Only the instructions were noted, no calculation was written down. Also, after the statement of the solution, no verification is carried out; instead we find a note stating that the solution is correct.

The title indicates that it is another example of an *ꜥḥꜥ*-problem. However, in this example, instead of adding a fractional part of the unknown quantity to itself, a multiple of it must be added. Consequently, the procedure to solve this problem differs from the two previous examples.

	2		D_1
	[1]		D_2
	9		D_3
1	$1 + 2 = 3$	1	$D_1 + D_2$
2	$9 \div 3 = 3$	2	$D_3 \div 1$

Rhind Mathematical Papyrus, Problem 50

Method of calculating *a circular* **area** *of 9 ḫt*
What is its amount as area?
You shall subtract its (i.e., the diameter's) $\bar{9}$ as 1,
while the remainder is 8.
You shall multiply 8 times 8.
It shall result as 64.
It is its amount as area: 64 *stꜣ.t*.

Calculation how it results: (9 *ḫt*)

.	9
its $\bar{9}$	1

subtraction from it, remainder: 8

.	8
2	16
4	32
\ 8	64

Its amount as area: 64 *stꜣ.t*.

Mathematics in Sumerian literature (i)

This humorously testy dialogue between two colleagues was popular in the scribal schools of Old Babylonian Nippur [ETCSL 5.1.3; Black et al. 2004, 277–80]. The older man begins by reminiscing idealistically about his days as a student, exhorting the younger man to pay attention and model himself on the strictures of his old teacher. The young man robustly responds that he has no need of such lecturing for he is now a fully competent scribe and administrator. The older man finally retracts his words, acknowledging that his younger colleague is now ready to become a teacher himself. They end by praising Nisaba, the patron goddess of scribes. Although neither mathematics nor any other school subject is explicitly mentioned here, we see the wide range of numerate tasks the scribe of a large household might have carried out, from allocating rations to the domestic staff to managing farm land and the people and animals working on it.

A Supervisor's Advice to a Younger Scribe

[1–8][*The supervisor speaks:*] 'One-time member of the school, come here to me, and let me explain to you what my teacher revealed. Like you, I was once a youth and had a mentor. The teacher assigned a task to me—it was man's work. Like a springing reed, I leapt up and put myself to work. I did not depart from my teacher's instructions, and I did not start doing things on my own initiative. My mentor was delighted with my work on the assignment. He rejoiced that I was humble before him and he spoke in my favor.

[9–15]'I just did whatever he outlined for me—everything was always in its place. Only a fool would have deviated from his instructions. He guided my hand on the clay and kept me on the right path. He made me eloquent with words and gave me advice. He focused my eyes on the rules which guide a man with a task: zeal is proper for a task, time-wasting is taboo; anyone who wastes time on his task is neglecting his task.

[16–20]'He did not vaunt his knowledge: his words were modest. If he had vaunted his knowledge, people would have frowned. Do not waste time, do not rest at night—get on with that work! Do not reject the pleasurable company of a mentor or his assistant: once you have come into contact with such great brains, you will make your own words more worthy.

[21–26]'And another thing: you will never return to your blinkered vision; that would be greatly to demean due deference, the decency of mankind. The heart is calm in

......, and sins are absolved. An empty-handed man's gifts are respected as such. Even a poor man clutches a kid to his chest as he kneels. You should defer to the powers that be and ...... —that will calm you.

[27–28]'There, I have recited to you what my teacher revealed, and you will not neglect it. You should pay attention—taking it to heart will be to your benefit!'

[29–35]The learned scribe humbly answered his supervisor. 'I shall give you a response to what you have just recited like a magic spell, and a rebuttal to your charming ditty delivered in a bellow. Don't make me out to be an ignoramus—I will answer you once and for all! You opened my eyes like a puppy's and you made me into a human being. But why do you go on outlining rules for me as if I were a shirker? Anyone hearing your words would feel insulted!

[36–41]'Whatever you revealed of the scribal art has been repaid to you. You put me in charge of your household and I have never served you by shirking. I have assigned duties to the slave girls, slaves, and subordinates in your household. I have kept them happy with rations, clothing, and oil rations, and I have assigned the order of their duties to them, so that you do not have to follow the slaves around in the house of their master. I do this as soon as I wake up, and I chivvy them around like sheep.

[42–49]'When you have ordered offerings to be prepared, I have performed them for you on the appropriate days. I have made the sheep and banquets attractive, so that your god is overjoyed. When the boat of your god arrives, people should greet it with respect. When you have ordered me to the edge of the fields, I have made the men work there. It is challenging work which permits no sleep either at night or in the heat of day, if the cultivators are to do their best at the field-borders. I have restored quality to your fields, so people admire you. Whatever your task for the oxen, I have exceeded it and have fully completed their loads for you.

[50–53]'Since my childhood you have scrutinized me and kept an eye on my behavior, inspecting it like fine silver—there is no limit to it! Without speaking grandly—as is your shortcoming—I serve before you. But those who undervalue themselves are ignored by you—know that I want to make this clear to you.'

[54–55][*The supervisor answers:*] 'Raise your head now, you who were formerly a youth. You can turn your hand against any man, so act as is befitting.'

[56–59][*The scribe speaks:*] 'Through you who offered prayers and so blessed me, who instilled instruction into my body as if I were consuming milk and butter, who showed his service to have been unceasing, I have experienced success and suffered no evil.'

[60–61][*The supervisor answers:*] 'The teachers, those learned men, should value you highly. You who as a youth sat at my words have pleased my heart.

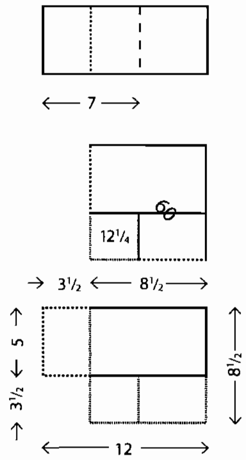
[62–72]'Nisaba has placed in your hand the honour of being a teacher. For her, the fate determined for you will be changed and so you will be generously blessed. May she bless you with a joyous heart and free you from all despondency ...... at whatever is in the school, the place of learning. The majesty of Nisaba ...... silence.

For your sweet songs even the cowherds will strive gloriously. For your sweet songs I too shall strive and shall ...... They should recognise that you are a practitioner (?) of wisdom. The little fellows should enjoy like beer the sweetness of decorous words: experts bring light to dark places, they bring it to closes and streets.'

73–74 Praise Nisaba who has brought order to ...... and fixed districts in their boundaries, the lady whose divine powers are divine powers that have no rival!

Geometrical algebra

In recent years Jens Høyrup has written extensively and authoritatively on Old Babylonian geometrical algebra, which comprises about half of the Old Babylonian corpus of mathematical problems [Høyrup 2002]. He has shown that underlying the apparently arithmetized texts are fundamentally concrete formulations that describe how to manipulate areas and lines in order to find unknowns. YBC 6967 is one of the simplest examples [MCT, text Ua; Høyrup 2002, 55–58]. The problem is to find a pair of mutual reciprocals knowing only the difference between them (and, by definition, that their product is 60: see BM 106444, on page 79). By visualizing the unknowns as the sides of rectangle of area 60 the rectangle can be manipulated into a gnomon and the original lengths found by completing the square:



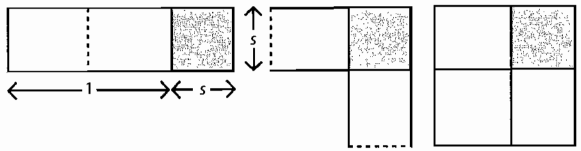
No such diagrams survive on the tablets themselves, but their use is inferred through a literal interpretation of the operations described. They explain, for instance, how two halves of a quantity can be manipulated independently of each other—whereas in symbolic algebra, a halved quantity is simply shrunk, rather than cut in two—and why each of four different verbs of multiplication is used in any particular context, as each has a specific geometrical or arithmetical function.

YBC 6967

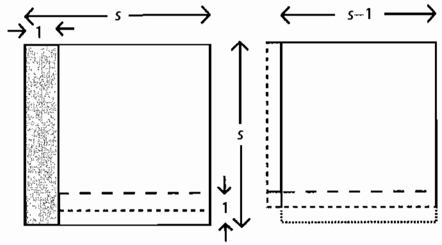
A reciprocal exceeds its reciprocal by 7. What are the reciprocal and its reciprocal? You: break in two the 7 by which the reciprocal exceeds its reciprocal so that 3;30 (will come up). Combine 3;30 and 3;30 so that 12;15 (will come up). Add 1 00, the area, to the 12;15 which came up for you so that 1 12;15 (will come up). What squares 1 12;15? 8;30. Draw 8;30 and 8;30, its counterpart, and then take away 3;30, the holding-square, from one; add to one. One is 12, the other is 5. The reciprocal is 12, its reciprocal is 5.

BM 13901, an unprovenanced set of twenty-four model solutions of problems in quadratic geometrical algebra, progresses from very simple scenarios about single squares to complex situations involving two squares or more [Thureau-Dangin 1936; Høyrup 2002, 50–77 passim]. It is analyzed in detail by [Høyrup 2001].

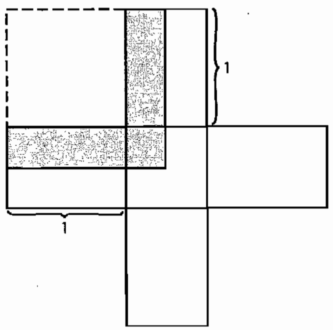
The diagrams of problems (i), (ii), and (xxxiii) below are modern attempts to interpret the scribe's words. In problem (i), the assignment is to find the side of a square given the sum of its area and side. In order to conceptualize this sum as a rectangle, the scribe converts the side into an area by multiplying it by 1, the "projection," and appending it to the square. As in YBC 6967, the solution is found through breaking the resultant rectangle at the midway point of the difference between the two sides, and rearranging the whole to create an L-shaped figure in order to generate a large square whose area can be calculated. The procedure is shown in this modern reconstruction:



Problem (ii), on the other hand, provides the difference between the area and side of the square before asking for the side. In this case, a "projection" is removed from the square to produce a rectangle. Again the difference between the two sides is cut in half and rearranged in a by now familiar configuration:



The following problems use similar cut and paste methods. In problem (xxiii), for instance, the four sides of the square are "projected" on to its area to make a cross-shaped figure of known area. Dividing this area by four produces an L-shaped figure that will then yield the answer in the usual way:



Interestingly, the final problem, (xxxiv), contains several calculation errors, perhaps because the scribe was losing concentration. The mistakes are eventually made to cancel each other out, presumably to produce the correct result without the bother of rewriting the procedure. It is in essence the same problem as (xiv) and those on Strasbourg 363 (page 110).

BM 13901

(i) I summed the area and my square-side so that it was 0;45. You put down 1, the projection. You break off half of 1. You combine 0;30 and 0;30. You add 0;15 to 0;45. 1 squares 1. You take away 0;30 which you combined from inside 1 so that the square-side is 0;30.

(ii) I took away my square-side from inside the area so that it was 14 30. You put down 1, the projection. You break off half of 1. You combine 0;30 and 0;30. You add 0;15 to 14 30. 14 30;15 squares 29;30. You add 0;30 which you combined to 29;30 so that the square-side is 30.

(iii) I took away a third of the area. I added a third of the square-side to inside the area so that it was 0;20. You put down 1, the projection. [You take away] a third of 1, the projection, [and] you multiply 0;40 by 0;20. You write down 0;13 20. You break [half of 0;20], the third which you added. You combine 0;10 and 0;10. You add 0;01 40 to 0;13 20. 0;15 [squares] 0;30. You take away [0;10 which you combined from inside 0;30] so that (it is) 0;20. The reciprocal of 0;40 [is 1;30. You multiply by 0;20 so that] the square-side is [0;30].

(iv) [I took away] a third [of the area]. I summed [the area and] my square-side (so that) it was 4 46;40. You put down [1, the projection]. You take away 0;20, a third of 1, the projection, and you multiply 0;40 by 4 46;40, and [you write down] 3 11;[06 40]. You break [in half] 1, the projection. You [combine] 0;30 and 0;30. You add [0;15 to 3 11;06 40]. 3 11;21 40 squares 13;50. [You take away 0;30] which you combined from [inside] 13;40 and <(it is) 13;20. The reciprocal of 0;40 is> 1;30. You multiply [by] 13;20 so that the square-side is 20.

(v) [I summed the area and my square-side and a third] of my square-side [so that it was 0;55]. You put down [1, the projection]. You add a third of [1, the projection to 1]: 1;20. [You combine] its half, 0;40, [and 0;40]. You add 0;26 40 to 0;55 and [1;21 40 squares 1;10. Take away 0;40 that you] combined from the middle of 1;10 so that the square-side is [0;30].